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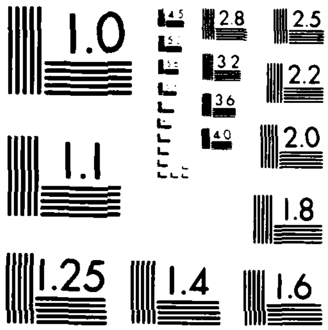

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

SCATTERED DATA INTERPOLATION
USING THIN PLATE SPLINES WITH TENSION

by

Richard Franke

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The equation of an infinite thin plate under the influence of point loads and mid-plane forces is developed. The properties of the function as the tension goes to zero or becomes large is investigated. This function is then used to interpolate scattered data, giving the user the parameter of tension to give some control over overshoot when the surface has large gradients. Examples illustrating the behavior of the interpolation function are given.

1. Introduction.

The underlying problem to be treated here is that of interpolation of scattered data. Given irregularly spaced points (x_k, y_k, f_k), $k=1,\ldots,N$, with the assumption that the (x_k, y_k) values do not all lie on a straight line, the problem is to construct a smooth function, $F(x,y)$, which takes on the values f_k at the points (x_k, y_k). This problem has received considerable attention over the last few years, and a great deal has been learned. Several surveys of the field are available, including Schumaker (1976), Barnhill (1977), and Franke (1982). The first two treat a wider class of approximations. The latter contains the results of testing a large number of algorithms for the problem. Despite the existence of many methods for construction of interpolating surfaces, a single scheme suitable for a wide range of dispositions of the independent variable data and for functions which have large gradients implied by the data has not yet been devised.

The problem of overshoot by surfaces in the presence of large gradients has been previously addressed by Nielson and Franke (1984) from the point of view of generalizing the univariate spline under tension. There the construction was based on an underlying triangulation of the independent variable data, and the surface reflected this as tension was increased. While potentially useful for user control of the surface, such a scheme clearly does not model a physical process, as does the univariate spline under tension (see Schweikert (1966) and references in Nielson and Franke). Another appoach to modeling such surfaces, based on the assumption that the surface actually

has a discontinuity has been investigated by Franke and Nielson (1983). Automatic detection of probable discontinuites was not considered.

The present paper is an attempt to investigate a scheme for representation of surfaces which incorporates the concept of tension in a way which models a physical process. The deflection of a thin plate under the imposition of point loads in the lateral direction and mid-plane forces (tension) will be developed. The theory of interpolation of scattered data by thin plate splines (TPS) under point loads is well known. Harder and Desmarais (1972) developed the equation from engineering considerations. This was followed by an elegant mathematical analysis and generalization by Duchon (1975,1976,1977) and Meinguet (1979,1979a). Further generalization to smoothing scattered data was given by Wahba and Wendelberger (1980).

The construction of the interpolation function, once the appropriate basis functions are determined, will parallel that of TPS and other methods based on the association of a basis function with each data point. Let the function $B_k(x,y)$ be associated with the point (x_k,y_k). The inclusion of some polynomial terms in the approximation may arise either in a natural way from the mathematics or physics of the problem, or from a desire to incorporate polynomial precision into the method. With linear terms the approximation then becomes

$$F(x,y) = \sum_{k=1}^{N} A_k B_k(x,y) + a + bx + cy \quad .$$

The coefficients in the approximation are determined by requiring

that the function interpolate, and that certain conditions on the coefficients be satisfied. The latter conditions are related to the polynomial terms, and imply the method has some polynomial precision (for linear functions, above). For TPS, linear functions are in the kernel of the functional being minimized. In addition, the constraint equations for TPS may be thought of as "equilibrium" conditions on the plate; the sums of the forces and moments must be zero. In the end, the coefficients are determined by the linear system

$$
\begin{aligned}
&\sum_{k=1}^{N} A_k B_k(x_i,y_i) + a + bx_i + cy_i = f_i \quad , \; i=1,\ldots,N \\
&\sum_{k=1}^{N} A_k = 0 \\
&\sum_{k=1}^{N} A_k x_k = 0 \\
&\sum_{k=1}^{N} A_k y_k = 0 \quad .
\end{aligned}
\tag{1}
$$

The development of the equation of deflection of a thin plate under the influence of point loads and mid-plane stresses will be developed from the engineering point of view in the next section. A discussion of the properties of the basis functions, and their behavior as tension goes to zero or becomes large will be given in Section 3. Some examples showing the effectiveness of the scheme in controlling overshoot and discussion of some problems of the scheme will be given in the Section 4. Finally, some speculations concerning a more elegant mathematization of the method will be discussed.

2. Basis Functions for Thin Plate Splines with Tension

The development of the basis functions for thin plate splines with tension (TPST) will approximately parallel that of Harder and Desmarais for surface splines (as they called thin plate splines). We begin with the equation of a thin plate subjected to lateral loads and mid-plane forces. This equation is found in books on plates and shells, for example, Timoshenko and Woinowsky-Krieger (1959,p. 379). In its general form the equation is

$$\nabla^4 W = \frac{1}{D}\left(q + N_x \frac{\partial^2 W}{\partial x^2} + N_y \frac{\partial^2 W}{\partial y^2} - N_{xy} \frac{\partial^2 W}{\partial x \partial y}\right) ,$$

where W is the lateral deflection, q is the lateral load, N_x, N_y, and N_{xy} are the mid-plane forces, and D depends on the properties of the plate material. Setting $N_y = N_x$ and $N_{xy} = 0$, one obtains an equation of the form

$$\nabla^4 W - \alpha^2 \nabla^2 W = p \quad ,$$

where is a tension parameter. The solution of the equation will be determined by a two step process. Let $V = \nabla^2 W$, then we have $\nabla^2 V - \alpha^2 V = p$.

The deflection of the plate, W_α, under a point load at the origin (the Green's function) will now be developed. Radial symmetry is assumed and converting the equation to polar coordinates under this assumption yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) - \alpha^2 V = \delta \; ,$$

where δ is the unit impulse functional. This is a modified

Bessel equation of order zero, and the two solutions of the homogeneous equation are $I_0(\alpha r)$ and $K_0(\alpha r)$. Among the boundary conditions to be applied are that the second derivatives of W_α tend to zero as $r \to \infty$. Thus I_0 is rejected and the solution is found to be $-(2\pi)^{-1}K_0(\alpha r)$. The factor $-(2\pi)^{-1}$ is obtained to satisfy the jump condition at the origin. Details are given in Stakgold (1979,p. 77). The remaining part of the problem now is to solve the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dW_\alpha}{dr}\right) = -(2\pi)^{-1}K_0(\alpha r) .$$

While no explicit elementary function is obtained, two integrations with application of the condition of finiteness at the origin and an arbitrary value, C, at the origin yield

$$W_\alpha(r) = -(2\pi)^{-1}\int_0^r t^{-1}\int_0^t sK_0(\alpha s)\,ds\,dt + C . \tag{2}$$

The lack of an explicit representation of the function is not a hindrance either theoretically or computationally. The details of computing with this basis function will be discussed in the next section.

By the process of superposition of point loads, and inclusion of an "equilibrium condition" for the plate, one obtains the representation

$$F(x,y) = \sum_{k=1}^{N} A_k W_\alpha(r_k) + a , \tag{3}$$

where $r_k^2 = (x-x_k)^2 + (y-y_k)^2$. The coefficients are obtained by solution of the system of equations corresponding to Eq. (1), but without the x and y terms in the approximation, and hence without

the last two conditions. The constraint equation requires that the sum of the loads (coefficients, A_k), be equal to zero. As will be seen in the next section, this condition could be derived by requiring that the surface become "flat" as $r \to \infty$. Because of the inclusion of the constant term in the approximation, one can take the value of C in (2) to be anything convenient. In my computations I took C = 0, but another value could lead to better numerical stability in the solution of the system for the A_k and a, especially for large values of tension. This was not investigated.

3. Properties of the Approximation

We first note that the lack of an elementary representation of the basis function is not a serious problem. The function can be easily approximated, either by numerical approximation of the integral formulation, above, or by considering the function as one of the components of the solution of a pair of ordinary differential equations. In the numerical examples given in a later section, the latter approach was used. Using the ODE solver, DVERK from the IMSL library, a table of values of the function was obtained over a suitable range for the problem at hand, and with a spacing which allowed linear interpolation for values at intermediate points to the desired accuracy (in my case, to essentially single precision accuracy on an IBM computer). To illustrate the behavior of the function for various values of the tension parameter, Figure 1 gives plots of the function for several parameter values. Here, each function has been normalized to have value 1 at r = 2. Multiplication of a

basis function by a constant has no effect on the overall approximation since the constant is accounted for in the coefficient. As the tension parameter is increased, the basis function W_α becomes more and more "pointed" at the origin. Because the equation of the thin plate with tension approaches the membrane equation (Laplace's equation) as tension gets large, and because the Green's function for the membrane equation is $\log r$, this is not surprising.

According to the differential equation the overall approximation (if not the individual basis functions) should approach the TPS when the tension is decreased to zero. However, this cannot be the case since the TPS includes linear terms in the approximation, while the TPST includes only a constant term. Nevertheless, we can show the basis function W_α approaches the basis function for the TPS as $\alpha \rightarrow 0$. Since the Green's function for the thin plate (the biharmonic equation) is a constant times $r^2 \log r$, which changes sign at $r = 1$, it is not obvious that this will happen. In order to show that this is the case, an expansion of K_o, $K_o(\alpha r) = -(\log(\alpha r) + \gamma)(1 + (\alpha r)^2/4 + O((\alpha r)^2)$ for small values of α (see Abramowitz and Stegun (1964), p. 375) is substituted into the expression (2). Here γ is Euler's constant. Upon integration of the first few terms, one obtains

$$W(r) = -(2\pi)^{-1} r^2 \log r + (r^2/(8\pi))(1 - \log(\alpha/2) - \gamma) + O(\alpha^2).$$

Since additive r^2 terms in the basis do not change the TPS (it is invariant with respect to the units used to measure distance), it is seen that as $\alpha \rightarrow 0$, the basis functions do approach those of the TPS.

The TPST is also invariant under change of the units used to measure distance, provided the tension parameter is also changed to the new units. If r is replaced with βr, and α with α/β, a change of the variables of integration in (2) to obtain the same integral form reveals that the basis function is multiplied by a constant, β^2.

Before considering the variational formulation of the TPST, the properties of the approximation as $r \to \infty$ will be investigated. Consideration of $W_\alpha(r)$ for large r $(r > R)$ shows that

$$W_\alpha(r)=W_\alpha(R) - RW_\alpha'(R)\log R + RW_\alpha'(R)\log r + (2\pi)^{-1}\int_0^r t^{-1}\int_0^t sK_0(\alpha s)\,ds\,dt \;.$$

The asymptotic approximation for large argument (see Abramowitz and Stegun (1964), p. 378) is $K_0(\alpha s) \sim (\pi/(2\alpha s))^{-1/2}\exp(-\alpha s)(1 + O(s^{-1}))$. Substitution of this into the integral above shows it is asymptotic to

$$(\pi/(2\alpha^3))^{-1/2}(R^{1/2}\exp(-\alpha R)\log r + C_R + O(\exp(-\alpha r)),$$

where C_R is a constant depending on R. It is then seen that

$$W_\alpha(r) = D_R + E_R \log r + O(\exp(-\alpha r)),$$

where D_R and E_R are constants depending on R. Thus $W_\alpha(r) = O(\log r)$. The interpolant (3) is

$$F(x,y) = \sum_{k=1}^{N} A_k W_\alpha(r_k) + a, \quad \text{with} \quad \sum_{k=1}^{N} A_k = 0.$$

Using $r_k^2 = (x - x_k)^2 + (y - y_k)^2 = r^2 - 2\rho_k r\cos\theta_k + \rho_k^2 = r^2(1 - 2(\rho_k/r)\cos\theta_k + \rho_k^2/r^2)$ for large r, one obtains

$$F(x,y) = \sum_{k=1}^{N} A_k (D_R + (E_R/2) \log [r^2(1-2(\rho_k/r)\cos\theta_k + \rho_k^2/r^2)]$$

$$+ a + O(\exp(-\alpha r))$$

$$= (E_R/2) \sum_{k=1}^{N} A_k [\log\ r^2 + \log(1-2(\rho_k/r)\cos\theta_k + \rho_k^2/r^2)]$$

$$+ a + O(\exp(-\alpha r))$$

$$= (E_R/2) \sum_{k=1}^{N} A_k [-2(\rho_k/r)\cos\theta_k + \rho_k^2/r^2 + O(r^{-2})]$$

$$+ a + O(\exp(-\alpha r))$$

$$= a + O(r^{-1}) .$$

Using the same ideas, it is easy to show that the first partial derivatives of f are $O(r^{-2})$ as $r \rightarrow \infty$.

Consideration of the variational form of the TPST gives the minimization property for these approximations. The functional is the expected generalization of that for univariate spline under tension, given by

$$\iint_{R^2} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \alpha^2 \left(\frac{\partial w}{\partial x}\right)^2 + \alpha^2 \left(\frac{\partial w}{\partial y}\right)^2$$

The derivation of the above yields an additional term which depends only on the data. Proof that this functional has finite value relies on the fact that the first derivatives are $O(r^{-2})$, going to zero sufficiently fast for the integrals to exist.

The above approach yields a method with polynomial precison for constants only. If the data all lie on a plane not parallel with the xy-plane, the resulting surface will not be a plane.

This arises from the application of tension in the horizontal direction. It is trivial to obtain linear precision by inclusion of the two linear terms, as indicated in (1). The physical analogue of this is not immediately apparent, but the resulting surfaces are more consistent with the surfaces obtained using TPS, and perhaps more satisfactory overall. Of course, the above functional cannot be finite if the linear terms are included in the approximation. We give examples of approximations computed both ways in the next section.

4. Numerical Examples

Three examples will be given which exhibit a variety of behavior, both good and bad. All of the examples are defined on the unit square, so the tension values in the three cases have the same units. With the exception of Figure 5, all approximations were computed with the inclusion of linear terms. Figure 5 was computed with only the constant term for purposes of showing a comparison.

The first example is a series of surfaces which all interpolate the same data, but with increasing tension. The data is from Nielson and Franke (1984, Table 2). There are 23 points. No tension results in overshoot in the vicinity of the sharp gradient. Figure 2 shows that as the tension is increased, the overshoot becomes less, and finally disappears. In this example the surface is generally well behaved as tension is increased.

A second example is also from the paper by Nielson and Franke (Table 3) and involves only eight points. Figure 3 shows the emergence of the sharp "points" of the basis functions at the

data points as tension increases, while away from the data the function remains very smooth, in contrast to the piecewise linear function over the triangulation approached by the surface in the reference. The behavior appears to be somewhat like one would imagine a rubber sheet would behave: too thick to be a membrane, but supporting little displacement except near the data.

The third example shows that while the use of tension may help to control overshoot, it may lead to undesirable behavior away from the sharp gradient. The data is taken from Franke and Nielson (1983, Table 1), has 33 data points, and is an extreme example since the parent surface is discontinuous. Figure 4 shows that as the tension is increased the approximation tends to look a bit like a plane with local sharp peaks and dips to achieve interpolation. Thus, while moderate values of tension result in improvement of the overshoot, as tension increases the effect of this localization degrades the surface. For comparison, Figure 5 was computed with the approximation including only the constant term. In part (a), where the tension has a moderate value, little difference is seen. In part (b), for a larger value of tension the character of the surface on the back "flat" part is seen to be very different, with the surface being pulled down to the data in Figure 4, while being pulled up to the data in Figure 5.

Other surfaces have been computed including only the constant term, in particular, all of the surfaces illustrated above. The differences between them are small, for the most part, the example given in Figure 5 being the extreme case investigated. The additional examples are shown in the appendix.

5. Conclusions

The dispacement of a thin plate with tension has been developed and applied to the problem of scattered data interpolation. It appears the scheme may give the user a useful way of controlling the behavior of the interpolating surface. In one variable some work has been accomplished toward determination of a suitable value of the tension parameter (see Lynch (1982)). In the present situation, there is no known way to automatically choose the tension parameter to achieve the proper amount of control. However, this is not in marked contrast to the whole problem of scattered data interpolation, where one should proceed cautiously until it is determined how one or more methods perform for data sets with the given configuration and features. Other issues, such as scaling of the data may alter the approximation in a significant way (e.g., see Breaker (1983)), and may be important here as well.

The work of Duchon resulted in the characterization of the TPS in terms of minimization of the thin plate functional in a certain reproducing kernel Hilbert space. It is almost certain that the TPST also minimizes the energy functional noted above in a similar space.

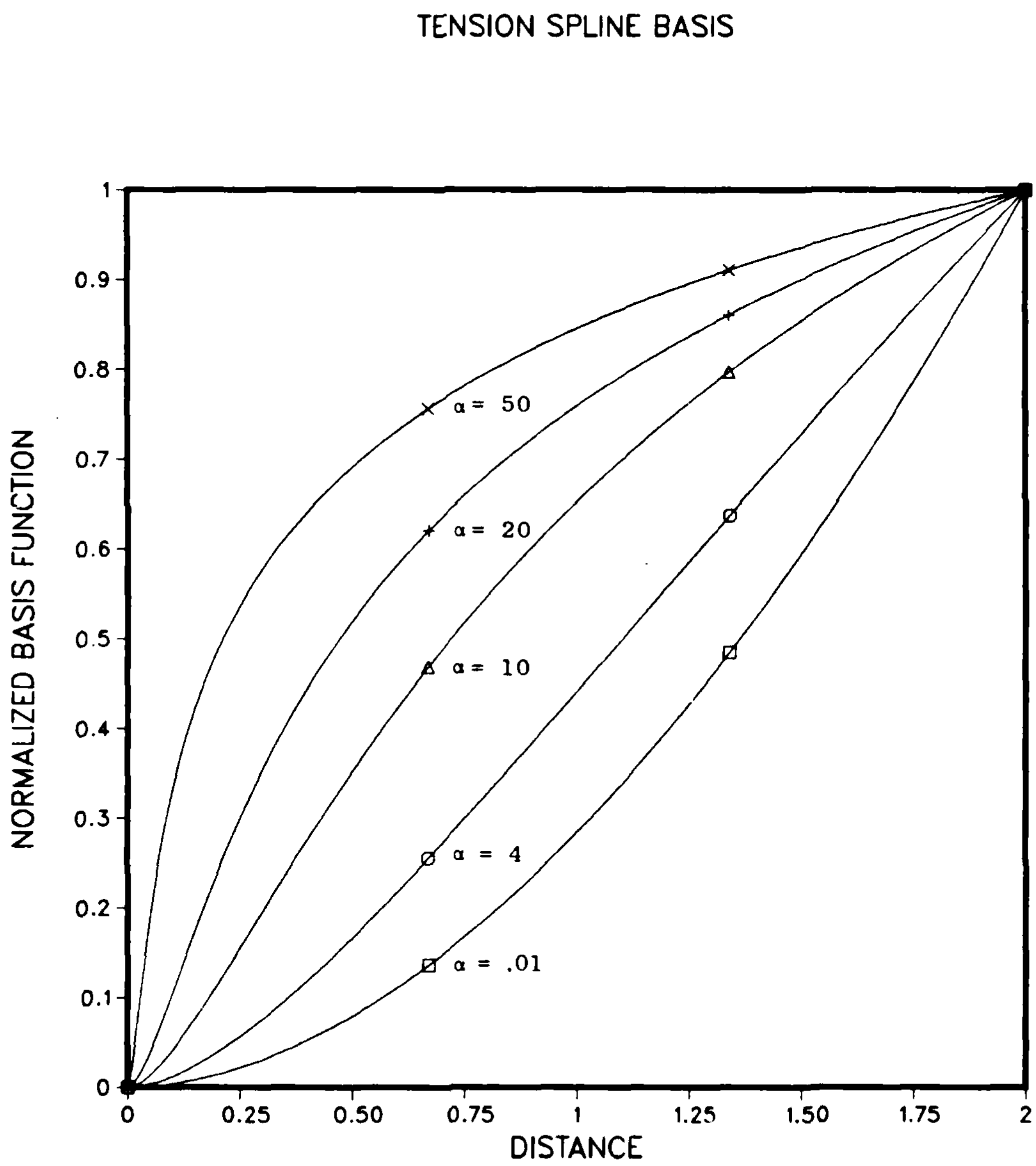
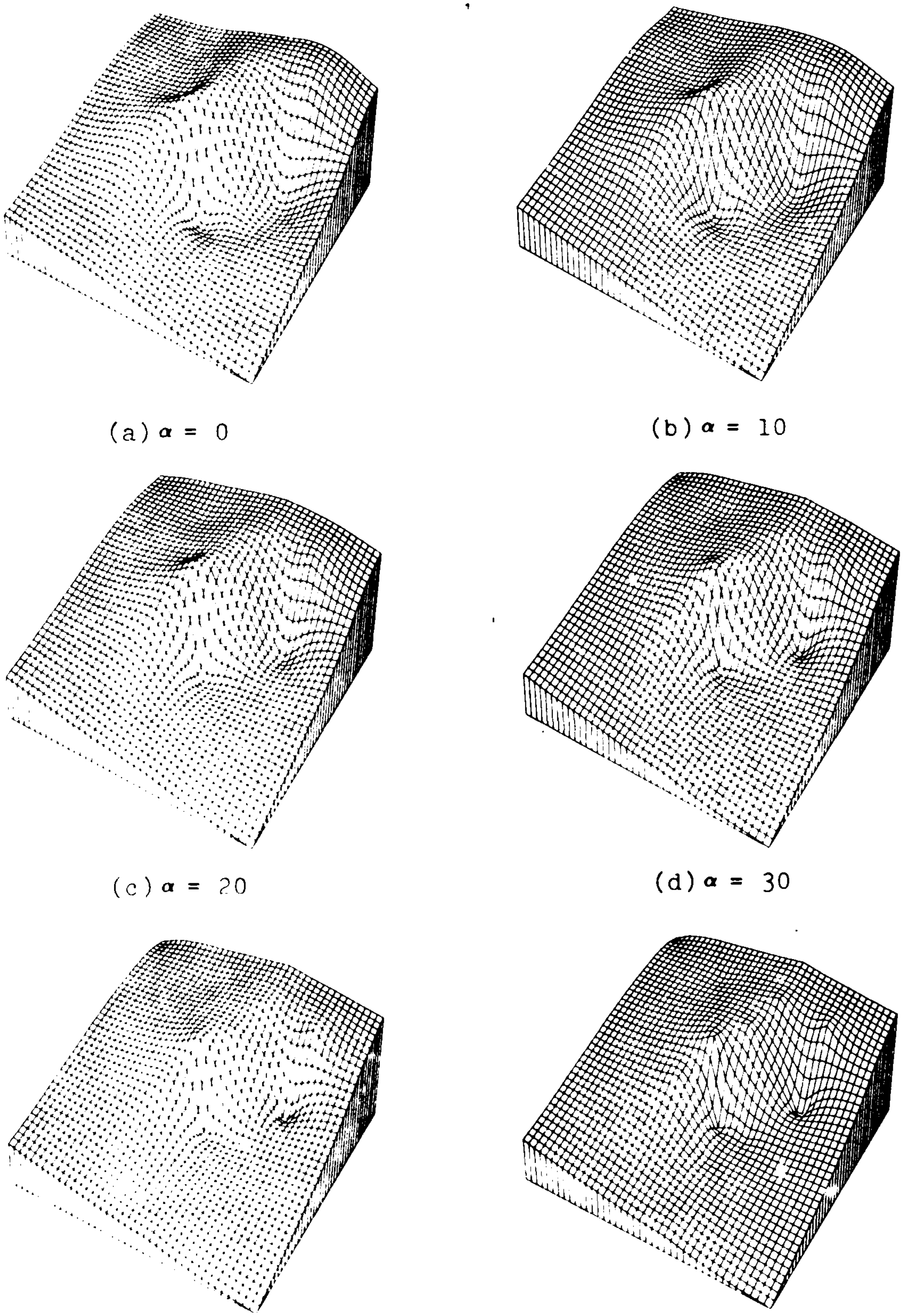

TENSION SPLINE BASIS
NORMALIZED BASIS FUNCTION
DISTANCE
α = 50
α = 20
α = 10
α = 4
α = .01
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.25
0.50
0.75
1
1.25
1.50
1.75
2

Figure 1



(a) α = 0

(b) α = 10

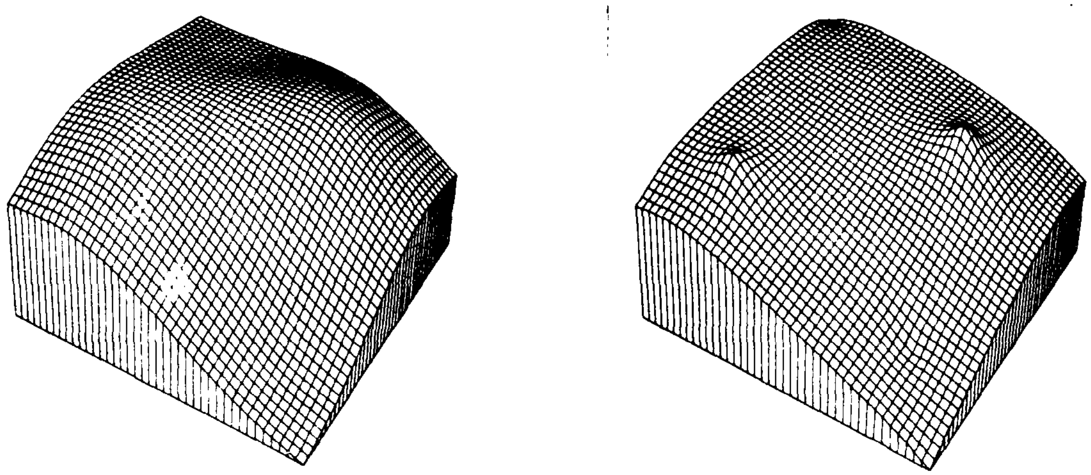
(c) α = 20

(d) α = 30

(e) α = 40

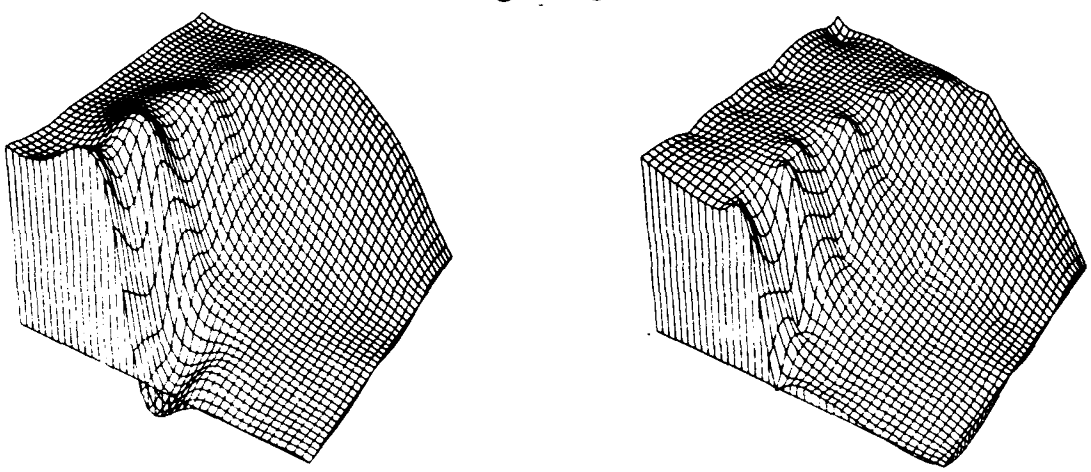
(f) α = 50

Figure 6



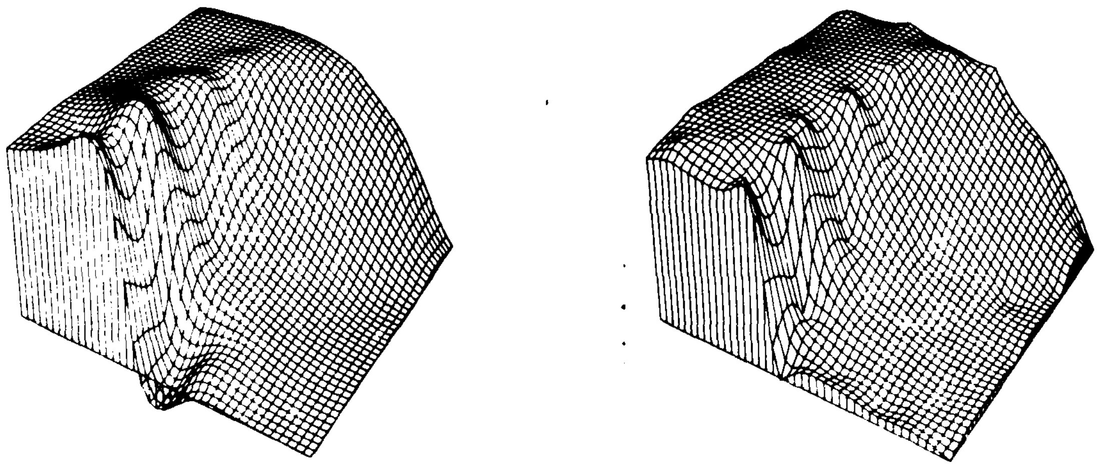
(a) α = 10 (b) α = 50

Figure 3



(a) α = 10 (b) α = 50

Figure 4



(a) α = 10 (b) α = 50

Figure 5

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Appendix

This appendix is given to illustrate more fully the behavior of the TPST as well as to document the differences between the two variants of it. The main content is a set of full page plots corresponding to a number of different data sets interpolated with various tension values, using the linear terms, on the left (even numbered pages, Figure n.a), and the same data interpolated using only the constant term, on the right (odd numbered pages, Figure n.b). This enables one to make quick comparisons between the two surfaces. Table 1 is a table of contents for the plots, identifying the Figure number in which the surface appears. For completeness I have included tables of the data, in Tables 2-6.

It will be seen that there are only small differences in the corresponding surfaces for small values of tension ($\alpha = 10$). In some cases the surfaces are very similar for large values of tension ($\alpha = 50$) as well, although in other cases the surfaces are significantly different, as in the surfaces shown in Figures 4 and 5. These same surfaces are shown in Figures 15a and 17a in this appendix, and it is seen that the more pleasing of the two is the surface including only the constant term. On the other hand, perusal of the surfaces shown in Figures 13 and 14 shows the more pleasing surface to be that including the linear terms. Figures 13a and 14a are the same surface as shown in Figure 2d and 2f. The other examples may show one or the other to be more pleasing; to a certain extent it is a personal choice.

It is apparent that neither of the schemes will be the better in all cases. There are good reasons for wanting such a method to have linear precision, and I have favored the scheme

incorporating the linear terms. Nonetheless, in certain applications it may well be advantageous to include only the constant term, and this is easy to justify. Thus, the user can decide based on individual mathematical preference, or on which scheme gives the more pleasing surface.

X	Y	F	X	Y	F
0.1375	0.9750	0.0000	0.4500	1.0375	0.0000
0.9125	0.9875	0.0000	1.0875	0.5500	0.0000
0.7125	0.7625	0.0000	0.5375	0.8000	0.0000
0.2250	0.8375	0.0000	-0.0375	0.7500	0.0000
-0.0500	0.4125	0.0000	0.1875	0.5750	0.0000
0.4750	0.6375	0.0000	0.7125	0.5500	0.0000
0.8500	0.4375	0.0000	1.0000	0.2625	0.0000
0.7000	0.3125	0.0000	0.5000	0.4625	0.0000
0.2750	0.4250	0.0000	0.1875	0.2625	0.2000
0.4500	0.2875	0.0000	0.5875	0.1250	0.0000
0.8125	0.1875	0.0000	1.0500	-0.0613	0.0000
0.4500	-0.0375	0.0000	0.1000	0.1125	0.0000
0.0500	-0.0500	0.0000			

Table 1: CARDINAL FUNCTION

X	Y	F	X	Y	F
0.0000	0.0000	0.4000	1.0000	0.0000	0.3000
0.0000	1.0000	0.2000	0.2000	0.3000	0.4000
0.3000	0.7000	0.4000	0.8000	0.2000	0.4000
0.8000	0.8200	0.1000	1.0000	1.0000	0.0000

Table 2: EIGHT POINT

X	Y	F	X	Y	F
0.0000	0.0000	0.3000	0.0000	0.6000	0.3600
0.0000	1.0000	0.3000	0.2400	0.0400	0.2910
0.1600	0.3200	0.3000	0.2000	0.6000	0.3300
0.2000	0.8000	0.3000	0.2000	1.0000	0.2400
0.4000	0.4000	0.3000	0.3600	0.5600	0.3000
0.3200	0.7600	0.0750	0.4000	1.0000	0.1500
0.4800	0.4800	0.3000	0.5200	0.5200	0.0750
0.5200	0.6400	0.0750	0.6000	0.0800	0.0210
0.6000	0.4000	0.2250	0.6000	0.8000	0.0750
0.6800	0.5600	0.0750	0.8000	0.2000	0.1500
1.0000	0.0000	0.1200	0.9600	0.6000	0.0600
1.0000	1.0000	0.0000			

Table 3: ROCKY MTN JRNL FNCTN

X	Y	F	X	Y	F
0.0400	0.0400	0.5000	0.4000	0.0400	0.5000
0.6000	0.0400	0.5000	0.8000	0.0000	0.5000
0.9600	0.0800	0.5000	0.0000	0.2800	0.5000
0.2000	0.2700	0.5000	0.4000	0.2000	0.5000
0.8800	0.2000	0.5000	0.1600	0.4000	0.5000
0.2800	0.3800	0.5000	0.4800	0.3600	0.5000
0.6800	0.3200	0.5000	0.8000	0.3700	0.5000
1.0000	0.3600	0.5000	0.4000	0.4400	0.3267
0.6400	0.4400	0.1960	0.8400	0.4200	0.0934
1.0000	0.4400	0.0000	0.7200	0.5600	0.0941
0.9200	0.6000	0.0222	0.4800	0.6400	0.1170
0.9600	0.7600	0.0040	0.6400	0.7600	0.0360
0.4000	0.8800	0.0150	0.8000	0.8800	0.0050
0.6000	1.0000	0.0000	1.0000	1.0000	0.0000
0.0800	0.4800	0.4911	0.0000	0.6000	0.4444
0.0400	0.8000	0.2778	0.0000	1.0000	0.0000
0.2800	1.0000	0.0000			

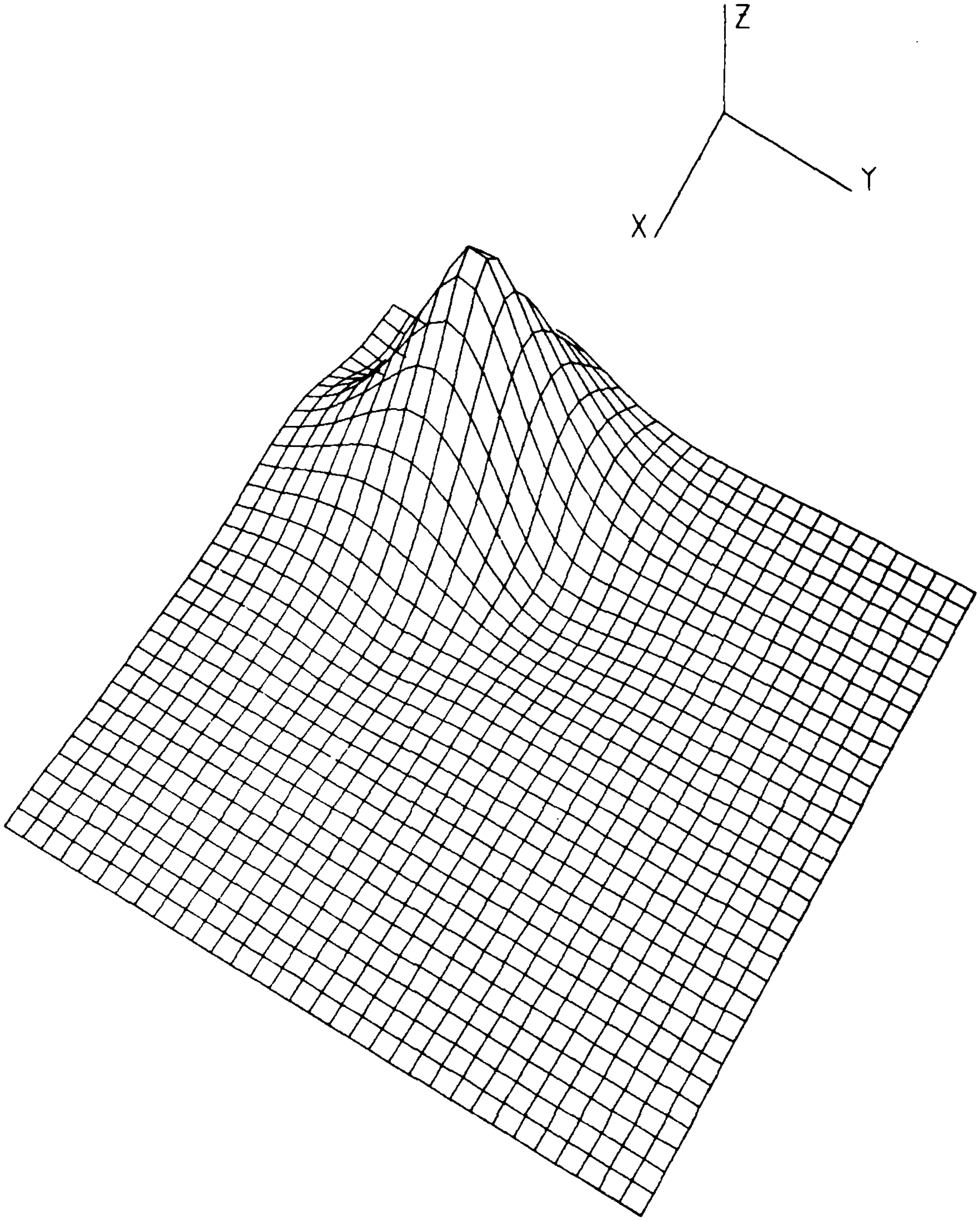
Table 4: FAULT LINE SIX

X	Y	F	X	Y	F
0.0400	0.0400	0.5000	0.4000	0.0400	0.5000
0.6000	0.0400	0.5000	0.8000	0.0000	0.5000
0.9600	0.0800	0.5000	0.0000	0.2800	0.5000
0.2000	0.2700	0.5000	0.4000	0.2000	0.5000
0.8800	0.2000	0.5000	0.1600	0.4000	0.5000
0.2800	0.3800	0.5000	0.4800	0.3600	0.5000
0.6800	0.3200	0.5000	0.8000	0.3700	0.5000
1.0000	0.3600	0.5000	0.4000	0.4400	0.3267
0.6400	0.4400	0.1960	0.8400	0.4200	0.0934
1.0000	0.4400	0.0000	0.7200	0.5600	0.0941
0.9200	0.6000	0.0222	0.4800	0.6400	0.1170
0.9600	0.7600	0.0040	0.6400	0.7600	0.0360
0.4000	0.8800	0.0150	0.8000	0.8800	0.0050
0.5000	1.0000	0.0000	1.0000	1.0000	0.0000
0.0800	0.4800	0.4911	0.0000	0.6000	0.4444
0.0400	0.8000	0.2778	0.0000	1.0000	0.0000
0.2800	1.0000	0.0000	0.0227	-0.0310	0.5000
0.0540	0.1587	0.5000	0.0217	0.2577	0.5000
0.0175	0.3414	0.5000	0.0019	0.4944	0.4876
-0.0510	0.5783	0.4559	0.0395	0.6993	0.3755
-0.0487	0.7470	0.3328	0.0316	0.9108	0.1377
-0.0419	0.9963	0.0062	0.1324	0.0501	0.5000
0.1090	0.0919	0.5000	0.1254	0.2593	0.5000
0.0935	0.3382	0.5000	0.0768	0.4171	0.4996
0.1453	0.7524	0.2070	0.0959	0.9147	0.1321
0.0627	0.6552	0.4095	0.2646	0.0293	0.5000
0.2392	0.0602	0.5000	0.2089	0.2669	0.5000
0.2767	0.3696	0.5000	0.1715	1.0000	0.0000
0.2267	0.5941	0.2212	0.2305	0.9046	0.0122
0.2426	0.9805	0.0005	0.3663	0.0397	0.5000
0.3858	0.0684	0.5000	0.3832	0.2390	0.5000
0.3179	0.3124	0.5000	0.3466	0.4903	0.2947
0.3777	0.5199	0.2490	0.3873	0.6445	0.1344
0.3813	0.8204	0.0347	0.3795	0.8938	0.0122
0.2804	0.9712	0.0010	0.4150	-0.0285	0.5000
0.4278	0.1561	0.5000	0.4200	0.2262	0.5000
0.4664	0.3175	0.5000	0.4856	0.3891	0.5000
0.4092	0.5085	0.2478	0.4793	0.6324	0.1222
0.4812	0.7511	0.0558	0.3978	0.8490	0.0239
0.4027	0.9979	0.0000	0.5849	-0.0272	0.5000
0.5730	0.1272	0.5000	0.6064	0.2709	0.5000
0.5014	0.3478	0.5000	0.5741	0.4259	0.2437
0.6107	0.6085	0.1036	0.5990	0.6734	0.0743
0.5381	0.7235	0.0613	0.6097	0.9242	0.0039
0.5026	1.0309	0.0008	0.6617	0.0256	0.5000
0.6428	0.0708	0.5000	0.6396	0.2008	0.5000
0.6704	0.3260	0.5000	0.7001	0.4891	0.1359
0.6334	0.5096	0.1531	0.6909	0.6698	0.0585
0.6896	0.7760	0.0270	0.6719	0.9366	0.0023
0.6838	1.0064	0.0000	0.7737	0.0285	0.5000
0.7635	0.1021	0.5000	0.7410	0.1937	0.5000
0.8259	0.3236	0.5000	0.7306	0.4714	0.1307
0.8087	0.6092	0.0507	0.8214	0.6685	0.0341
0.7291	0.8023	0.0184	0.8077	0.8477	0.0078
0.8171	1.0512	0.0008	0.8425	0.0380	0.5000
0.8684	0.0902	0.5000	0.8367	0.2083	0.5000
0.9419	0.3318	0.5000	0.8478	0.4336	0.0848
0.8600	0.5910	0.0407	0.9176	0.6307	0.0195
0.8596	0.8145	0.0084	0.9280	0.9042	0.0011
0.8513	0.9696	0.0002	1.0450	-0.0121	0.5000
[illegible]	0.1334	0.5000	0.9858	0.2696	0.5000
[illegible]	0.3795	0.5000	1.0129	0.4396	-0.0071
[illegible]	0.5044	0.0146	1.0020	0.6941	-0.0003
[illegible]	0.7460	-0.0040	1.0415	0.8682	-0.0012
[illegible]	0.9801	0.0000	1.0000	0.5678	0.0000

Table 5: FAULT LINE SEVEN

		Tension = α (=TN)		
Data Set	Table	10	30	50
Cardinal	1	6	7	8
8 Point	2	9	10	11
Rocky Mtn J F	3	12	13	14
Fault Line 6	4	15	16	17
Fault Line 7	5	18	19	20

Table 6: Figure number in which surface plot appears



25 PT CARDINAL FUNCTION 61984 1437
THIN PLATE WITH TENSION TN = 10.0

Figure 6.a

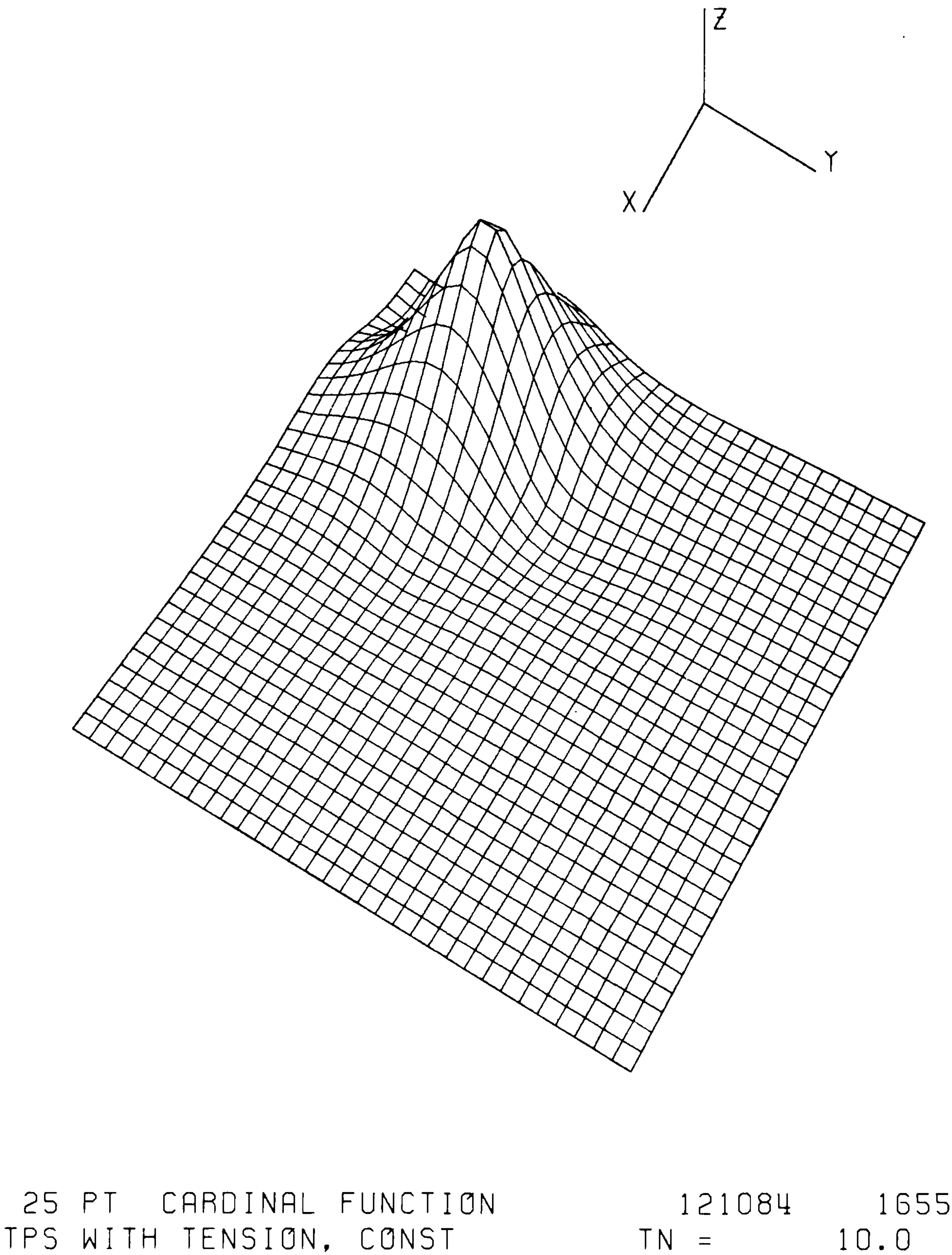

Z
Y
X
25 PT CARDINAL FUNCTION 121084 1655
TPS WITH TENSION, CONST TN = 10.0

Figure 6.b

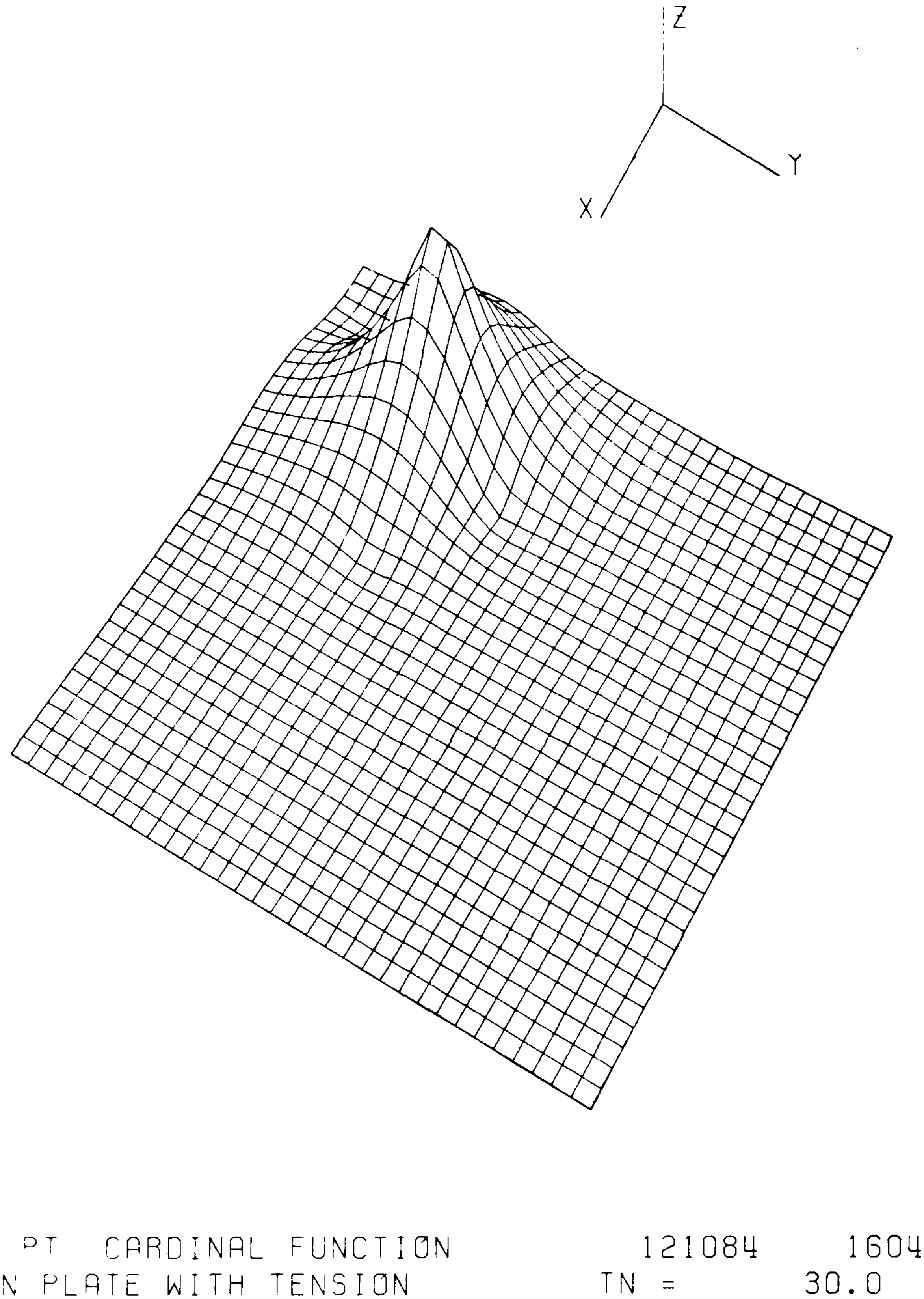
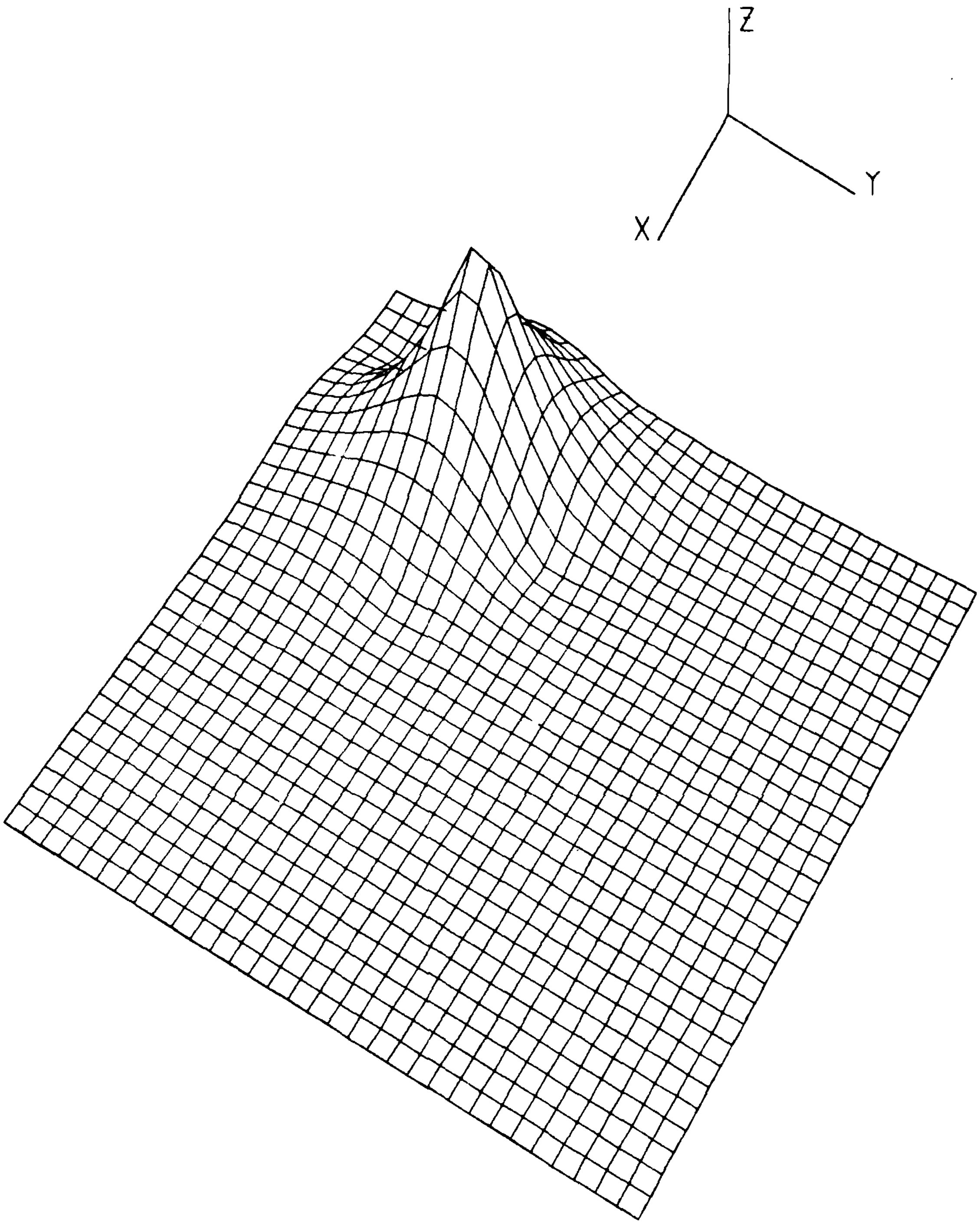
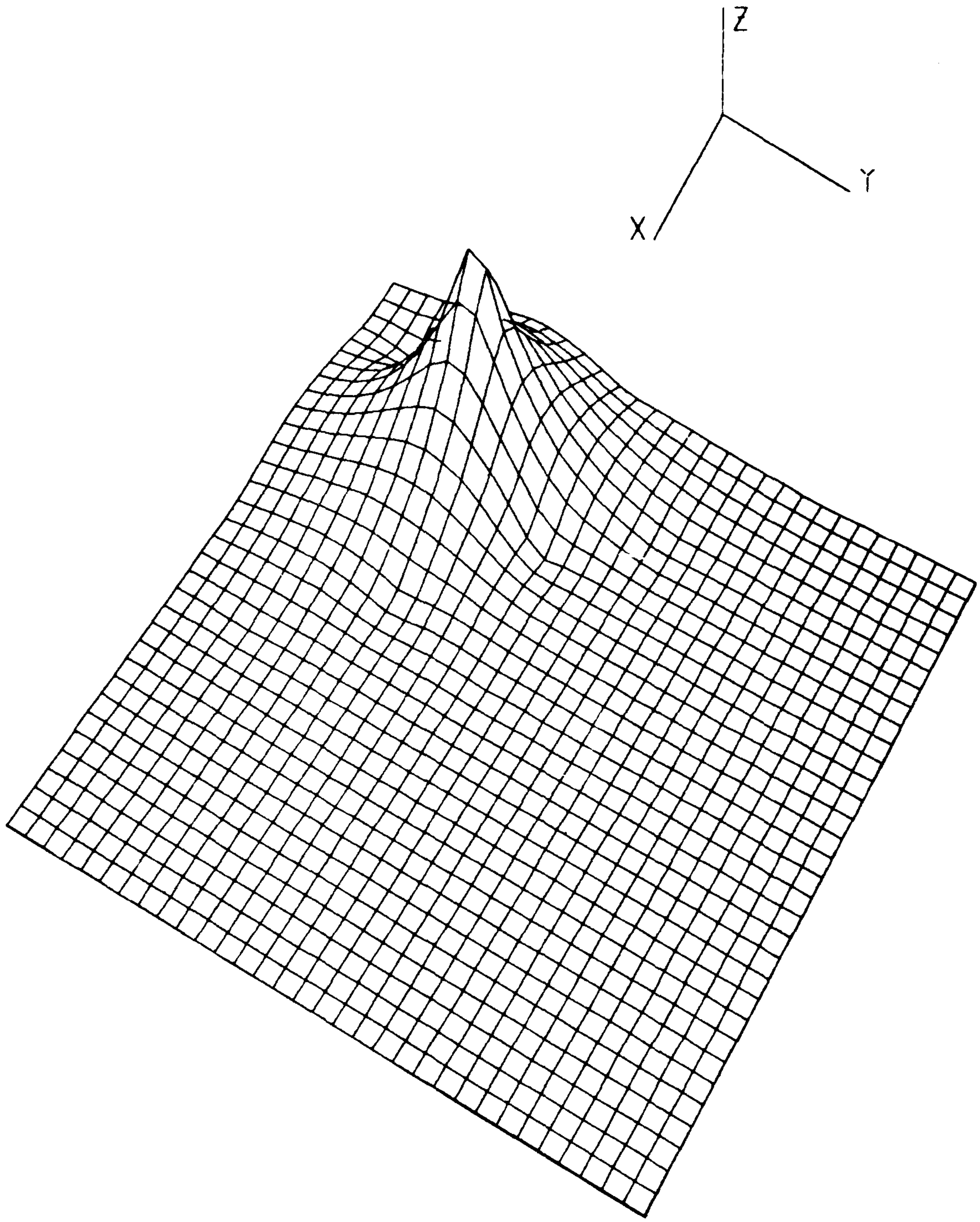

Z
Y
X
25 PT CARDINAL FUNCTION 121084 1604
THIN PLATE WITH TENSION TN = 30.0

Figure 7.a



25 PT CARDINAL FUNCTION 121184 1158
TPS WITH TENSION, CONST TN = 30.0

Figure 7.b



25 PT CARDINAL FUNCTION 61984 1437
THIN PLATE WITH TENSION TN = 50.0

Figure 8.a

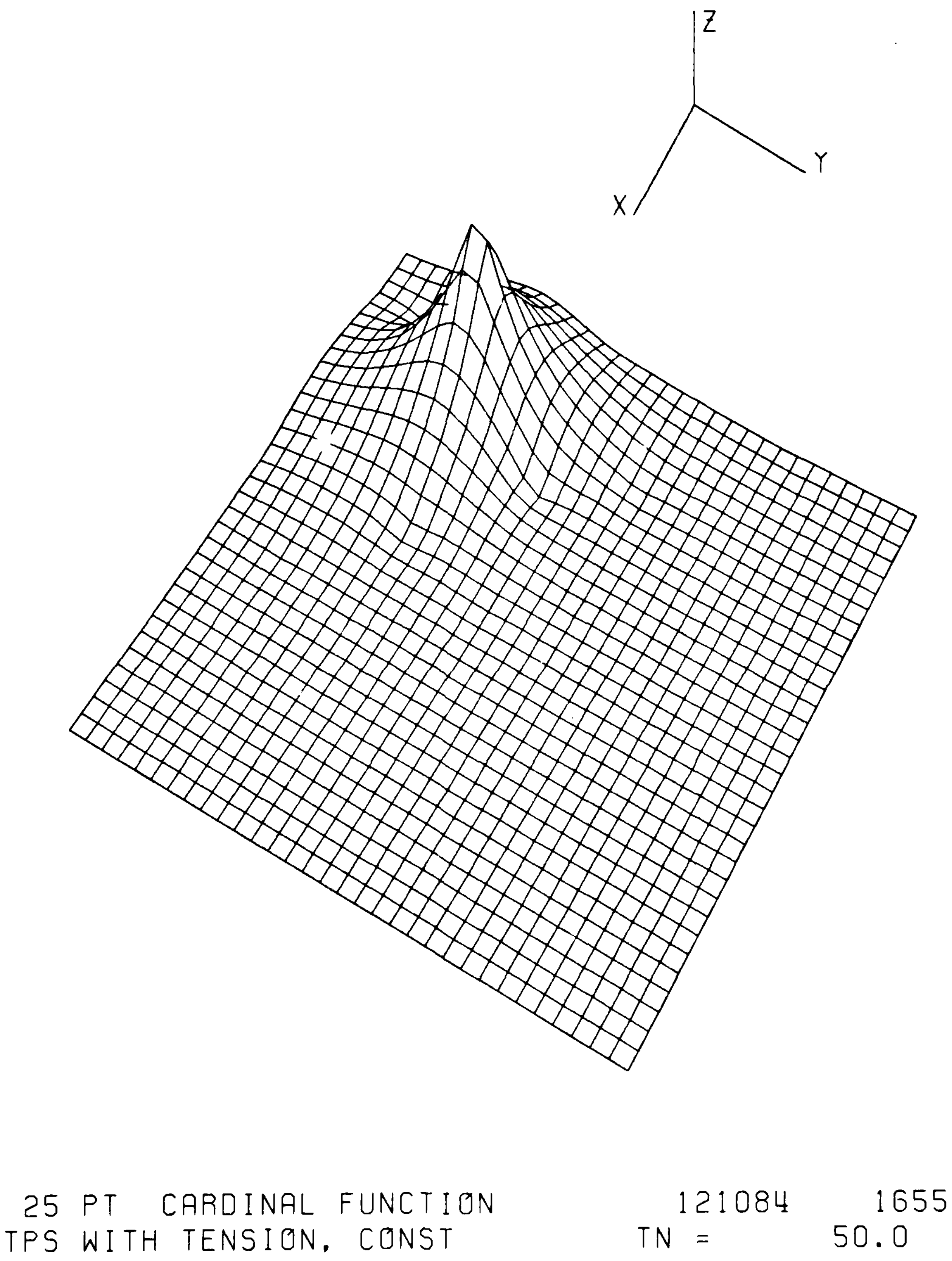

Z
Y
X
25 PT CARDINAL FUNCTION 121084 1655
TPS WITH TENSION, CONST TN = 50.0

Figure 8.b

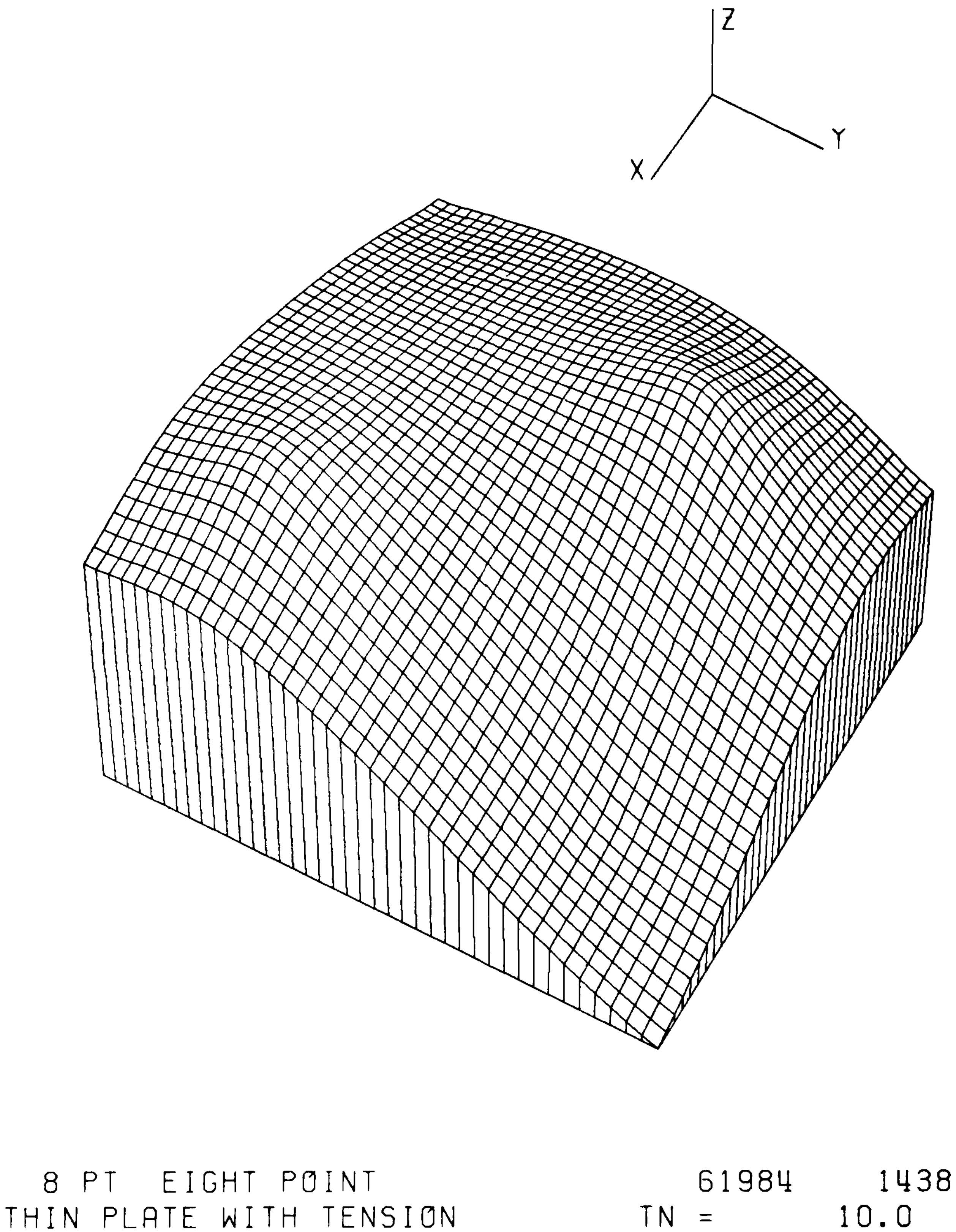

Z
Y
X
8 PT EIGHT POINT 61984 1438
THIN PLATE WITH TENSION TN = 10.0

Figure 9.a

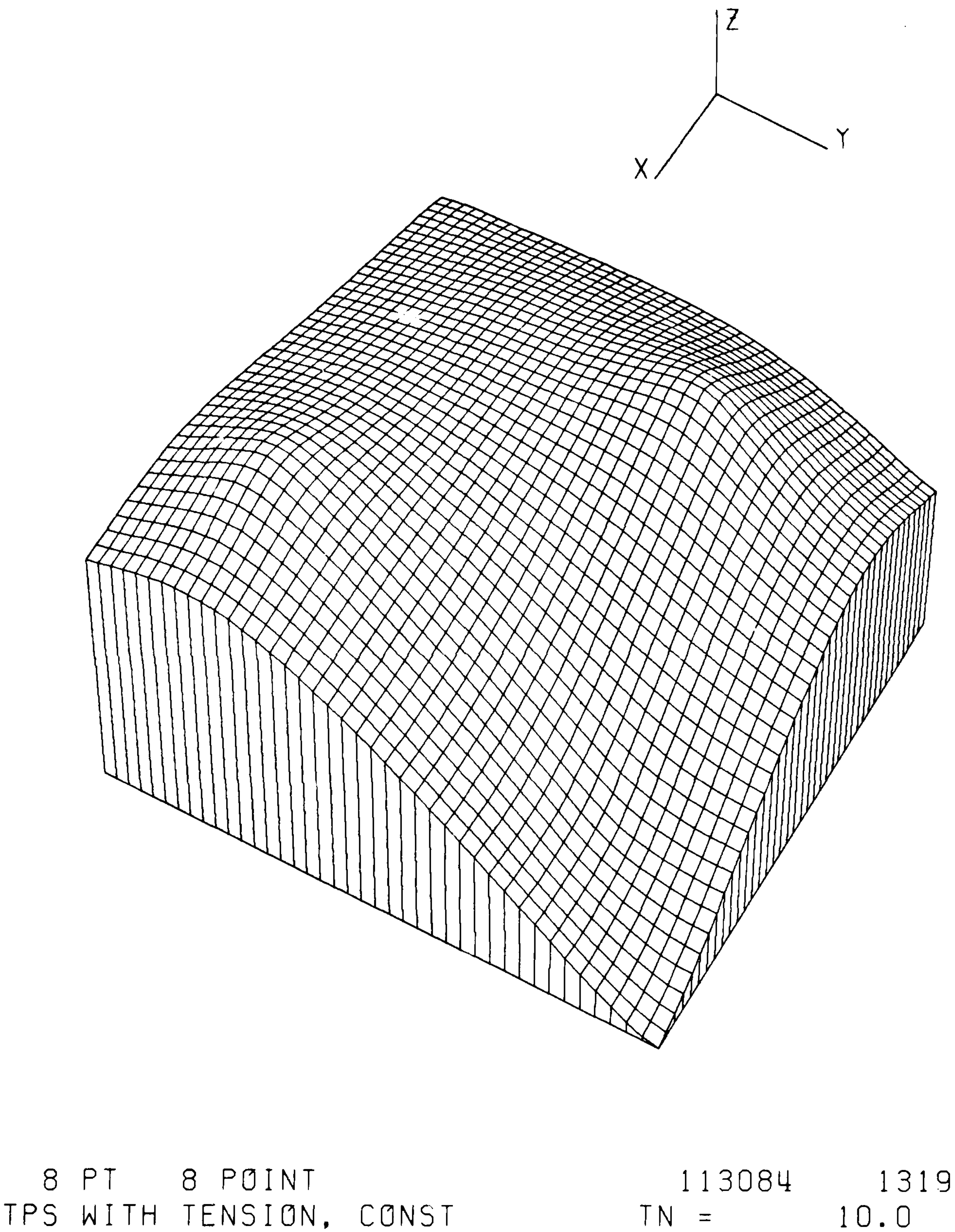


Figure 9.b

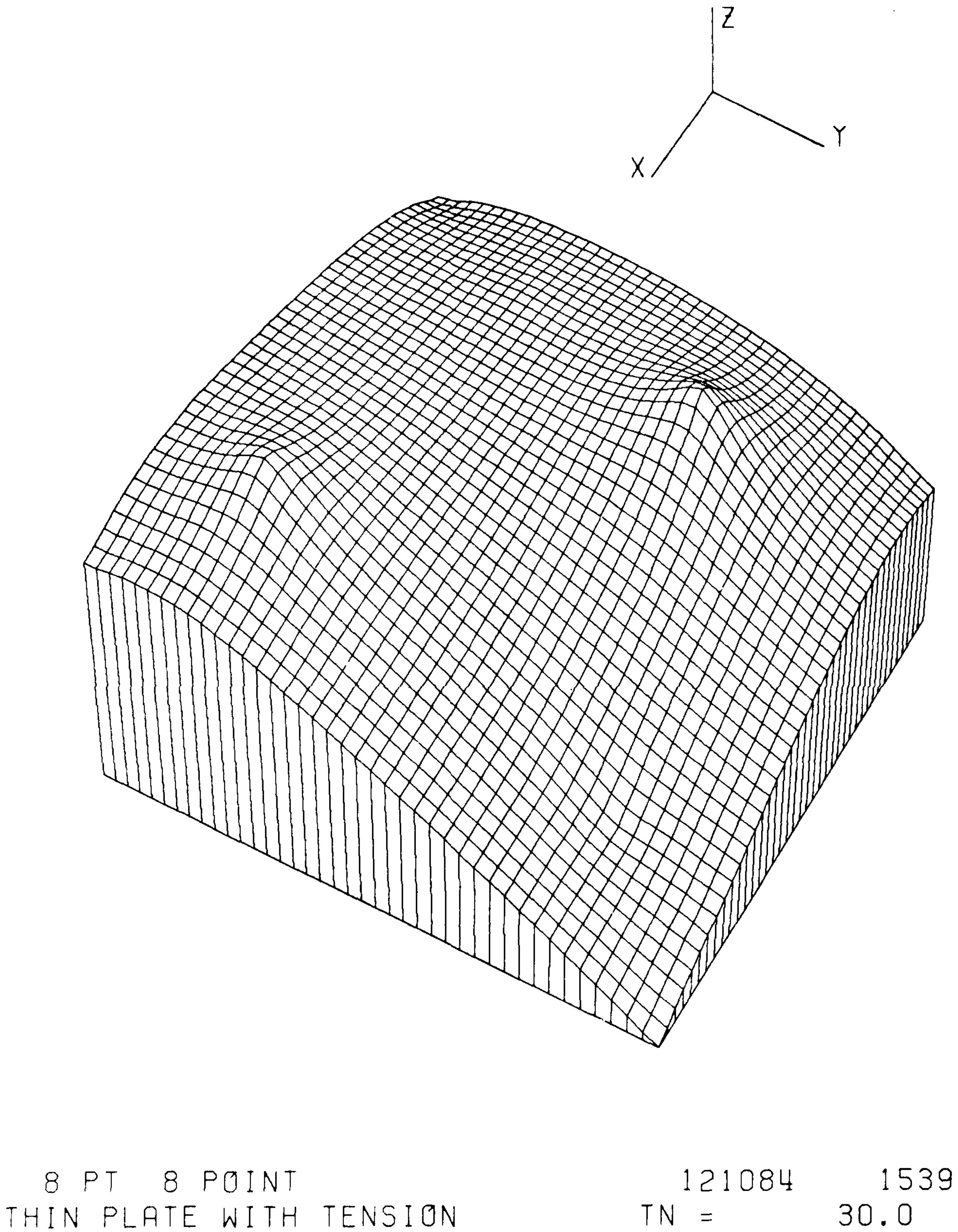


Figure 10.a

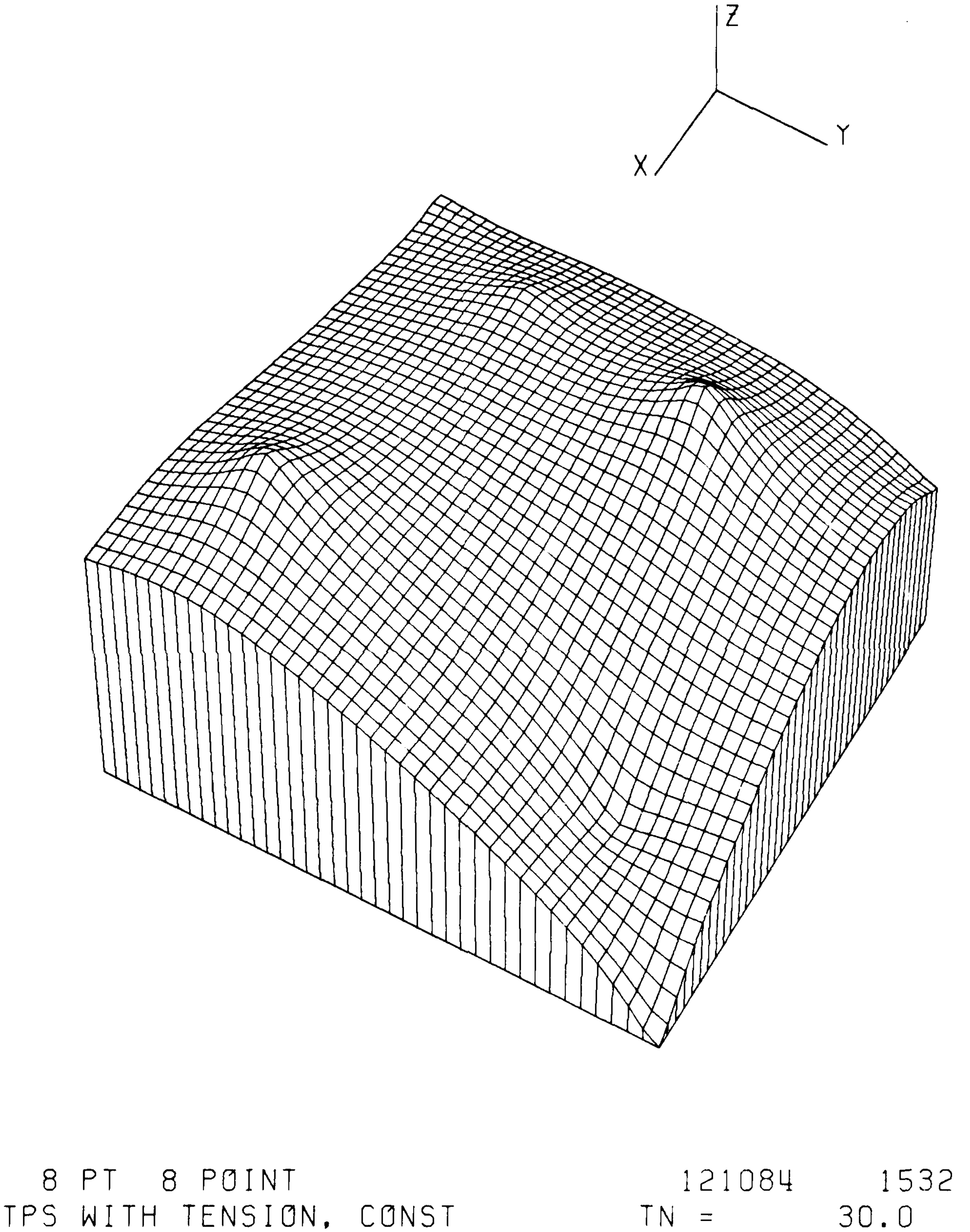
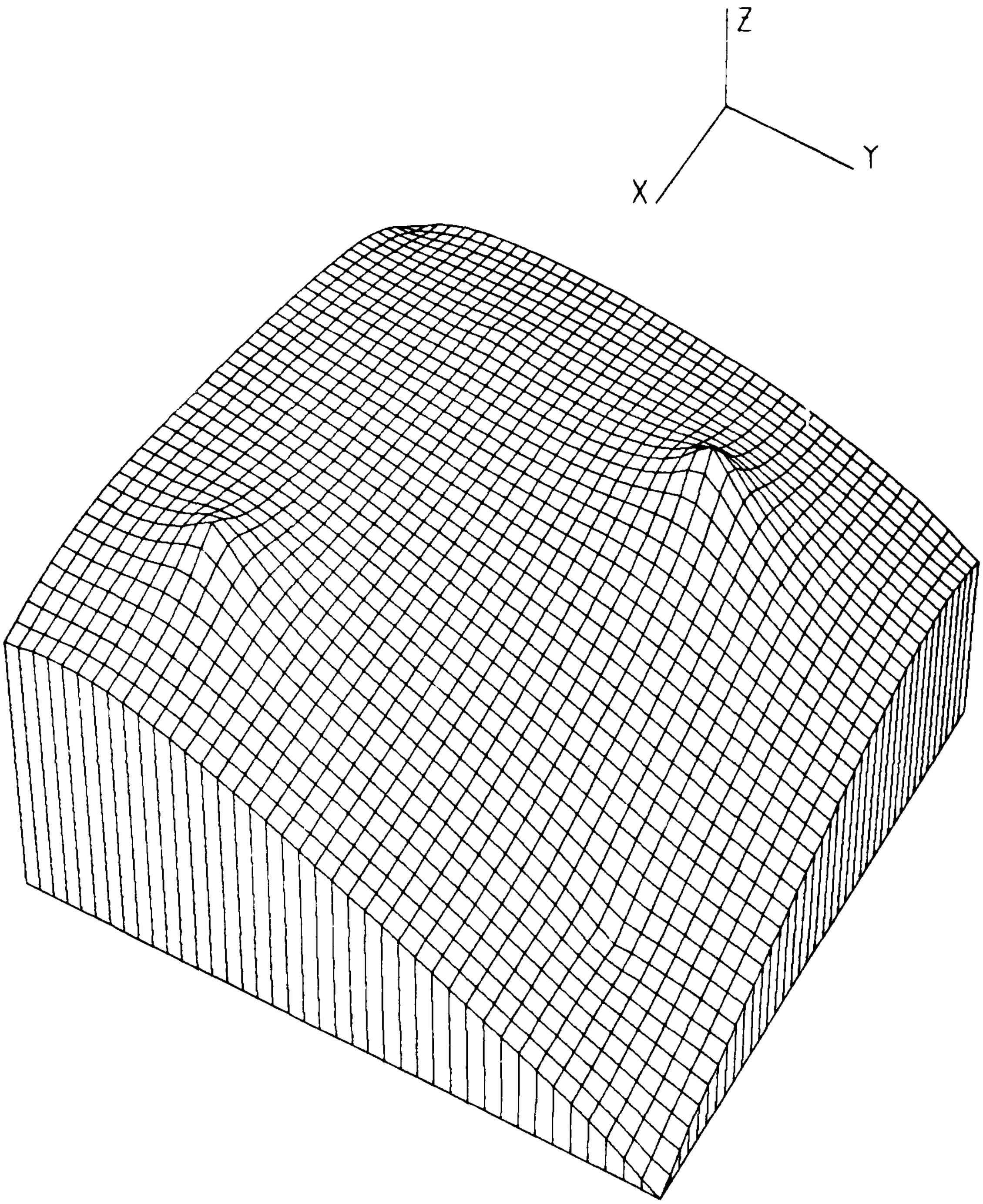


Figure 10.b



8 PT EIGHT POINT 61984 1438
THIN PLATE WITH TENSION TN = 50.0

Figure 11.a

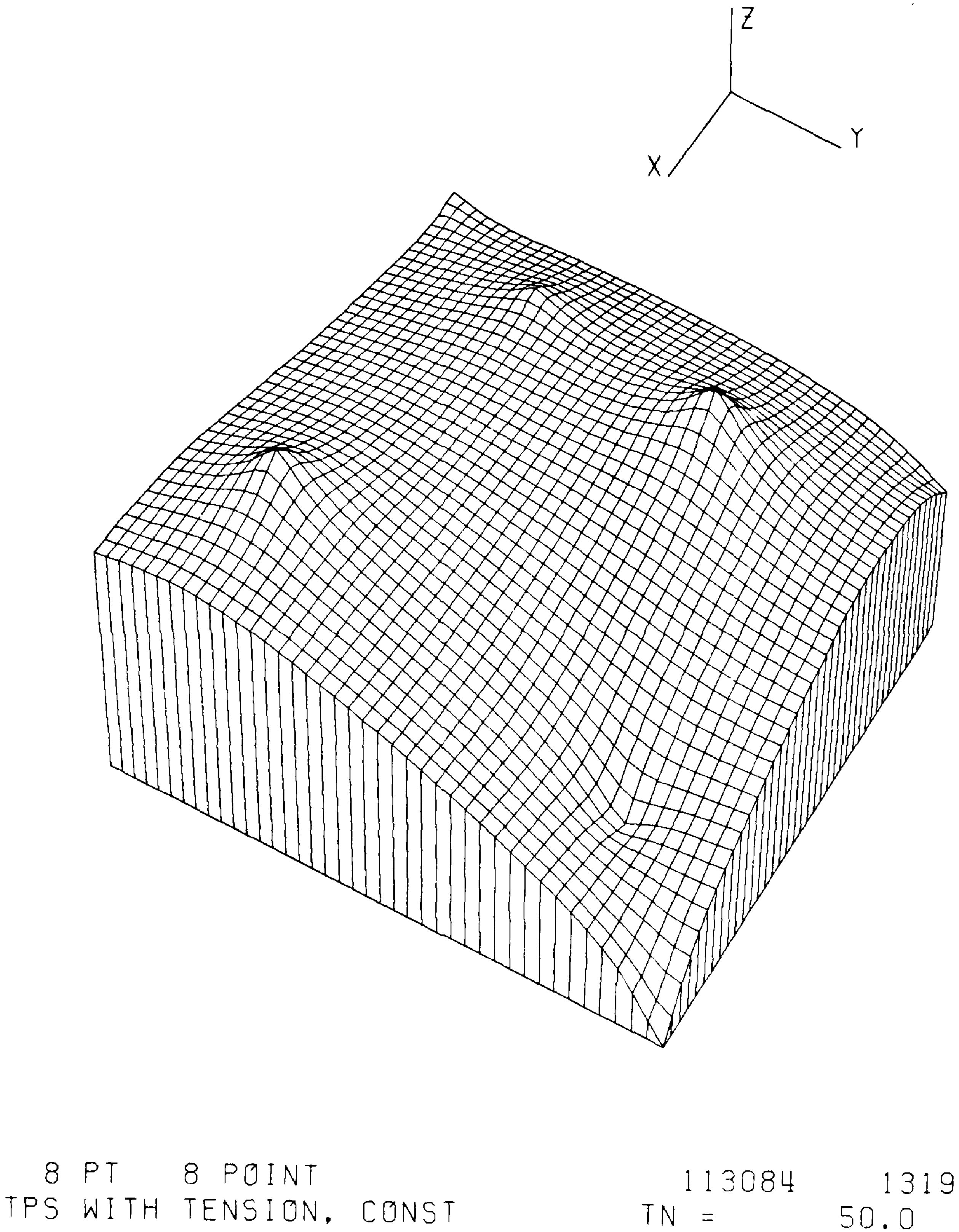
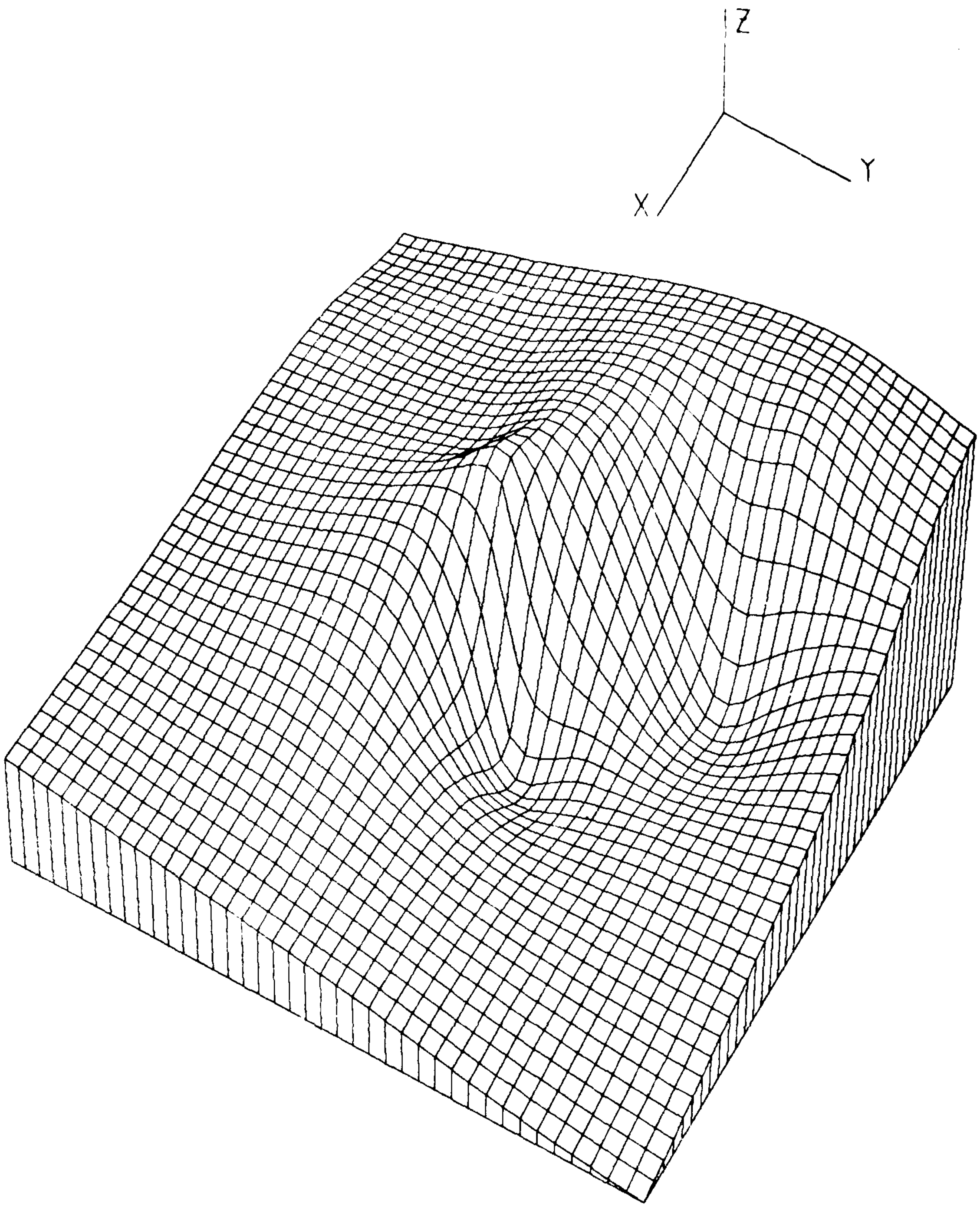
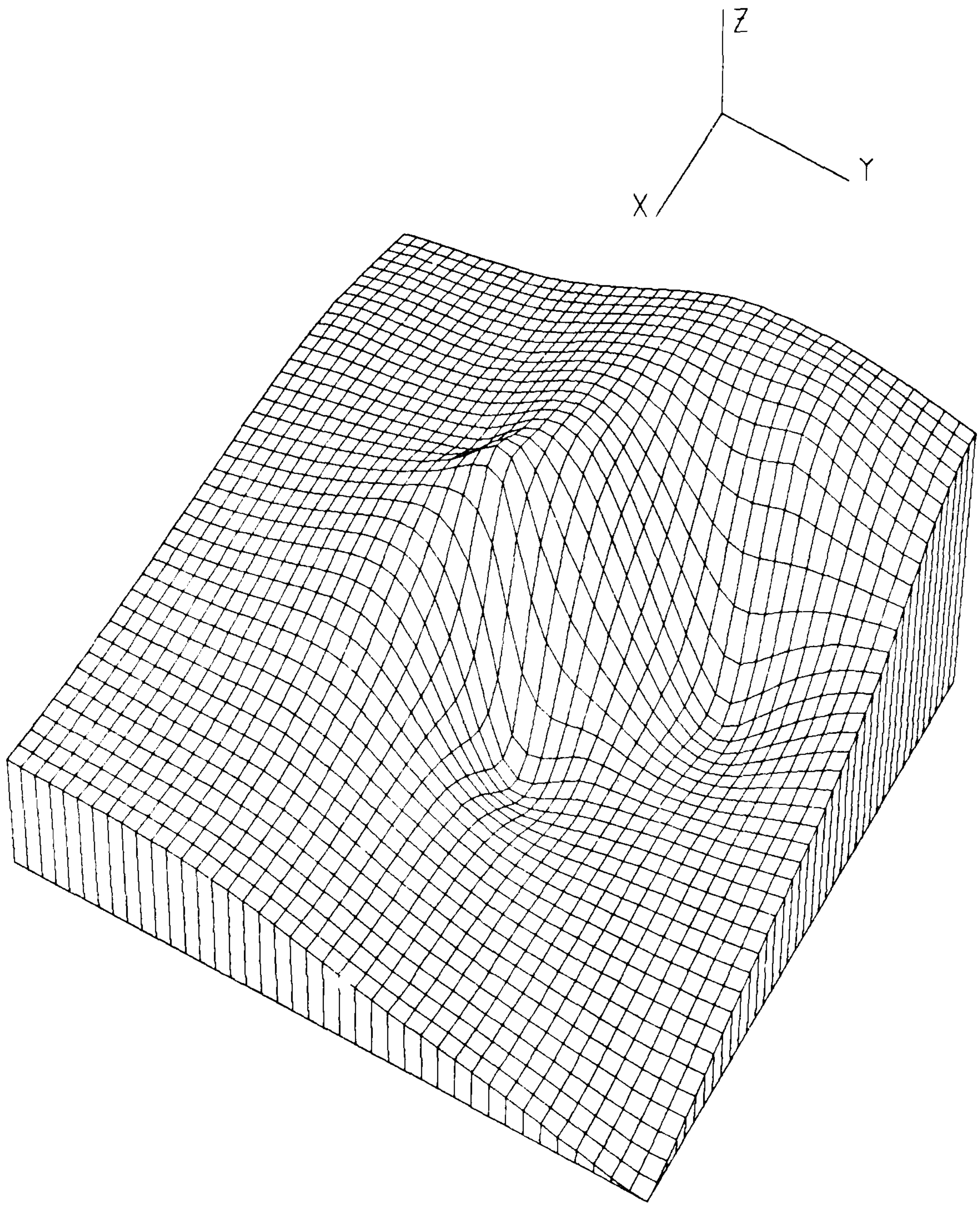


Figure 11.b



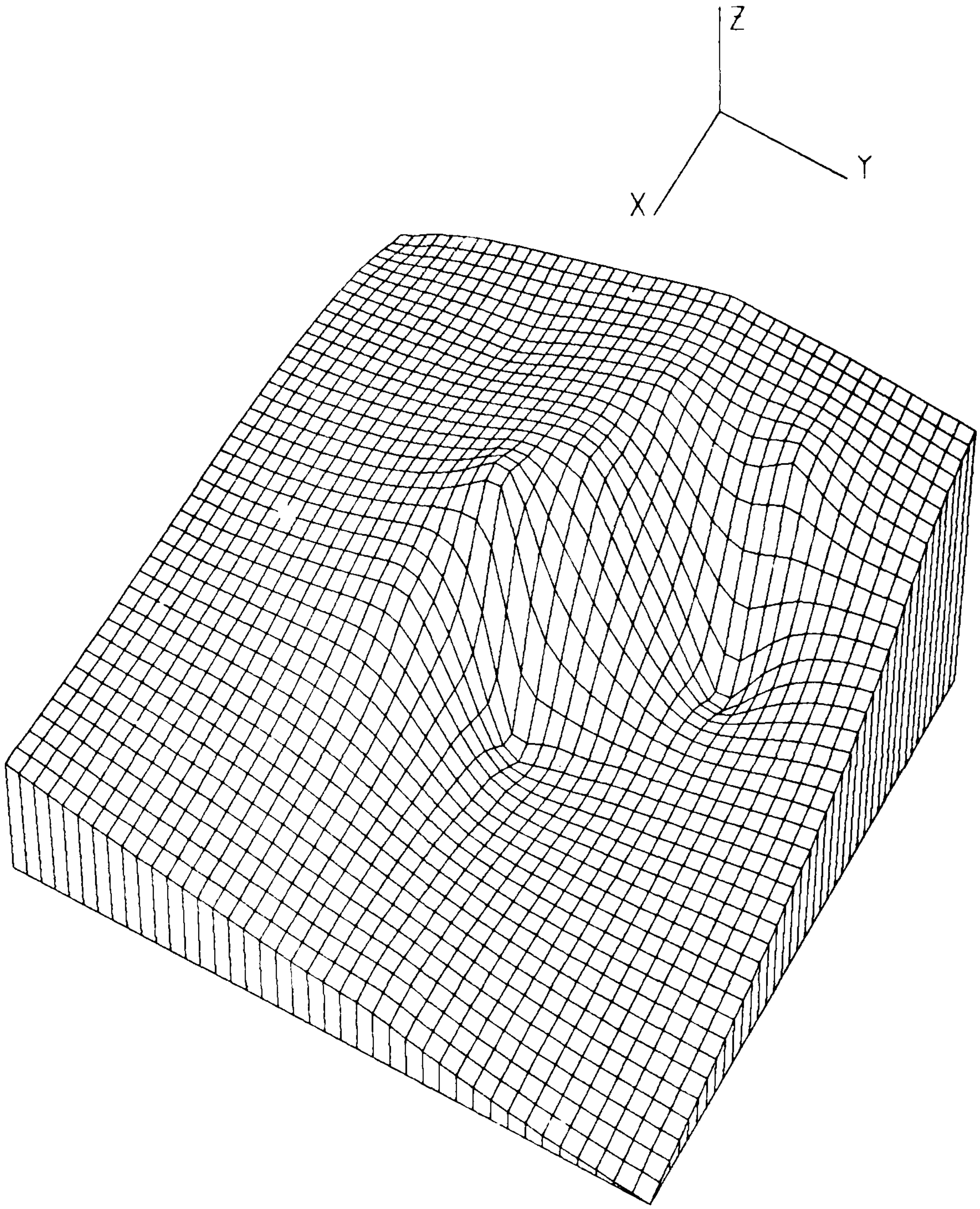
23 PT RØCKY MTN JRNL FNCTN 61984 1438
THIN PLATE WITH TENSIØN TN = 10.0

Figure 12.a



23 PT ROCKY MTN JRNL FNCTN 113084 1232
TPS WITH TENSION, CONST TN = 10.0

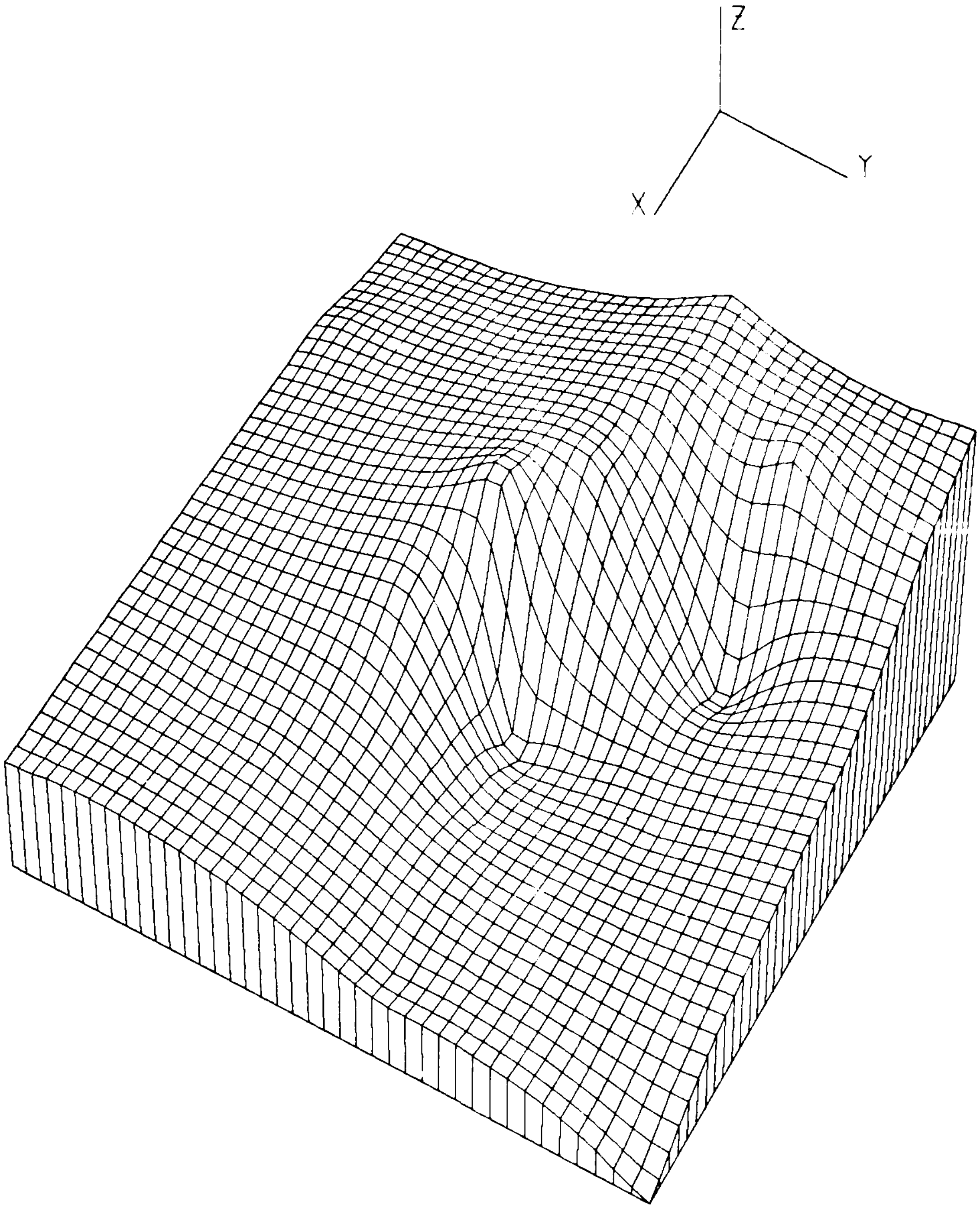
Figure 12.b



23 PT ROCKY MTN JRNL FNCTN 62184 827

THIN PLATE WITH TENSION TN = 30.0

Figure 13.a



23 PT ROCKY MTN JRNL FNCTN 120184 1431
TPS WITH TENSION, CONST TN = 30.0

Figure 13.b

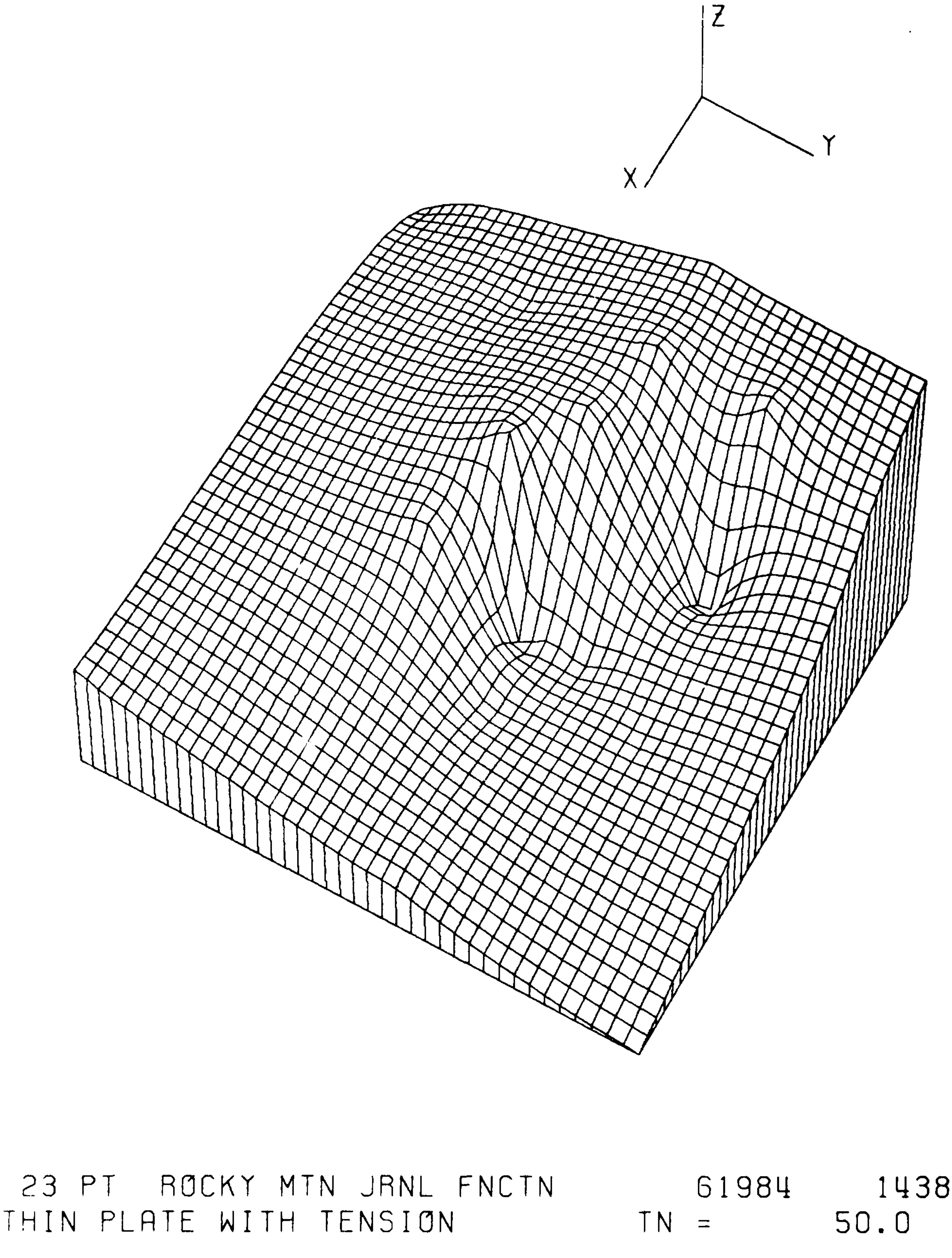
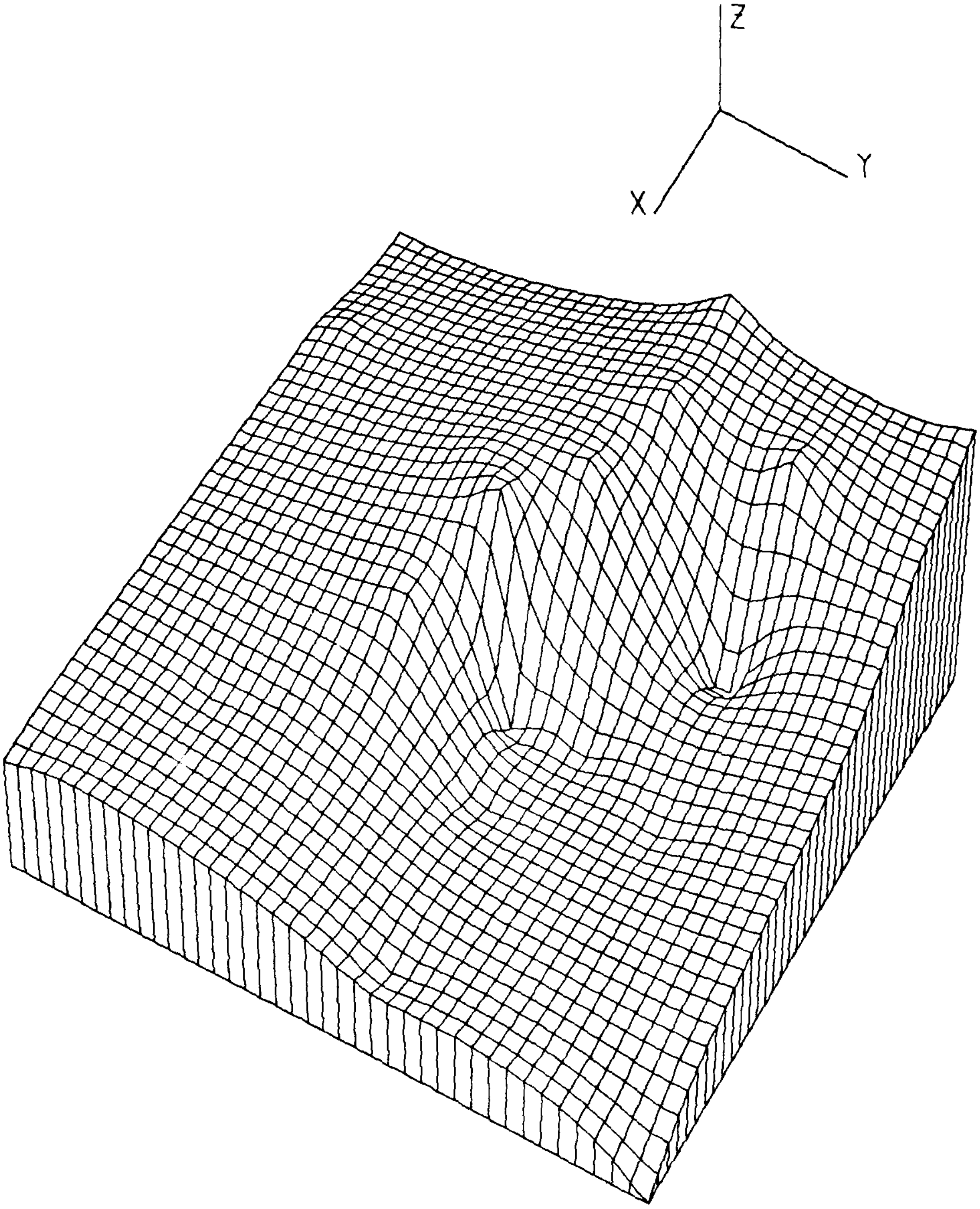

Z
Y
X
23 PT ROCKY MTN JRNL FNCTN 61984 1438
THIN PLATE WITH TENSION TN = 50.0

Figure 14.a



23 PT ROCKY MTN JRNL FNCTN 113084 1232
TPS WITH TENSION, CONST TN = 50.0

Figure 14.b

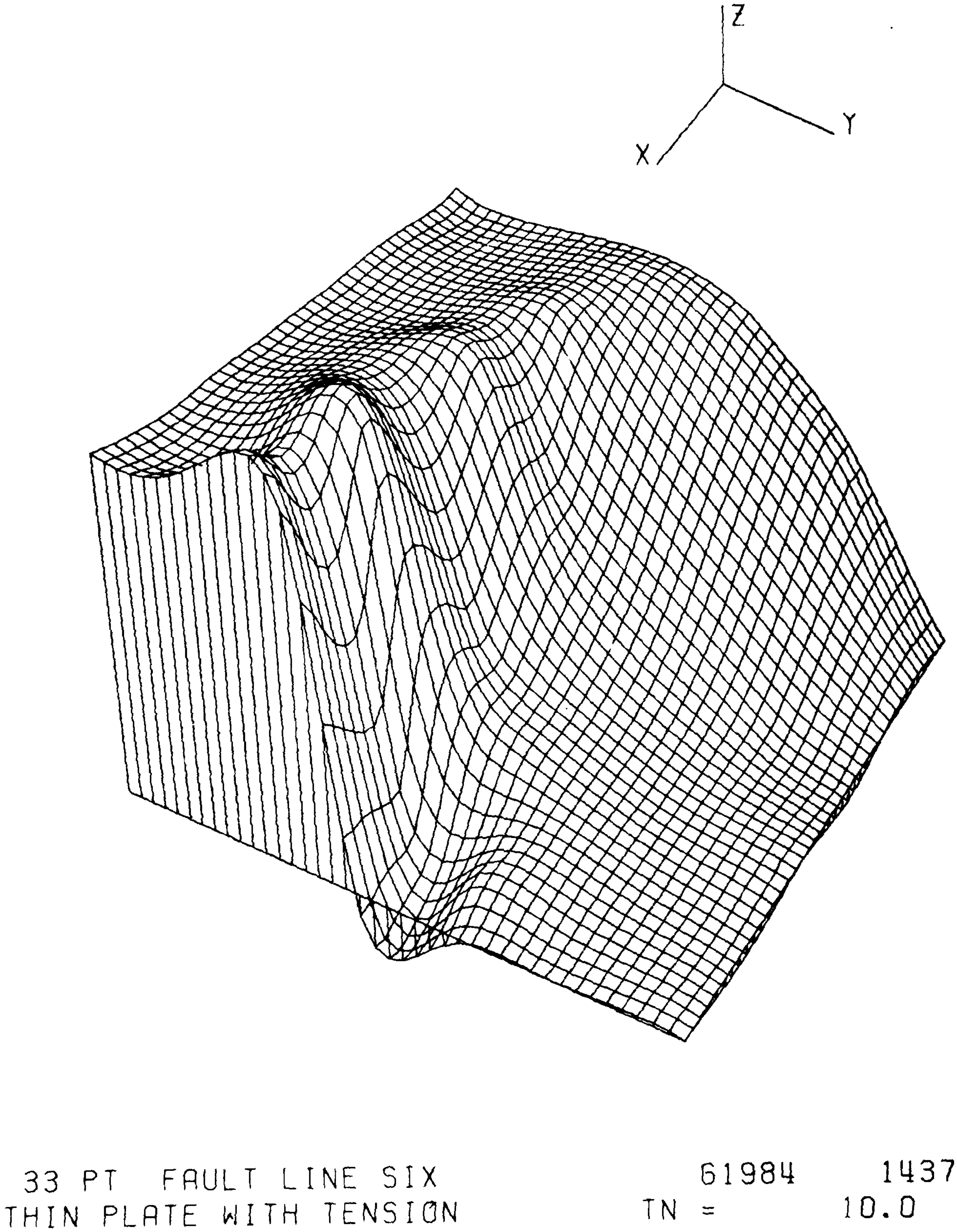

Z
X
Y
33 PT FAULT LINE SIX
THIN PLATE WITH TENSION
61984 1437
TN = 10.0

Figure 15.a

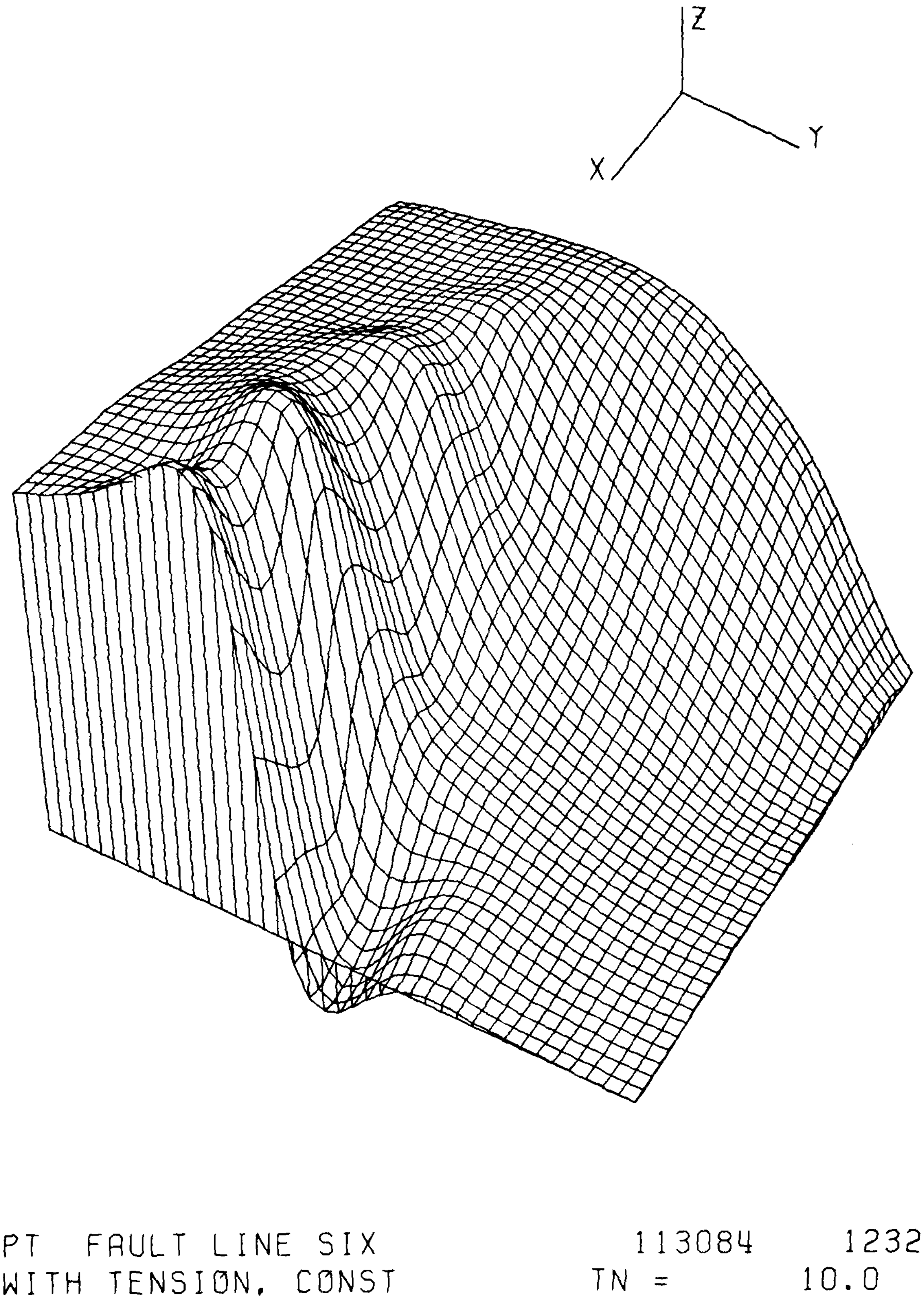

Z
X
Y
33 PT FAULT LINE SIX 113084 1232
TPS WITH TENSION, CONST TN = 10.0

Figure 15.b

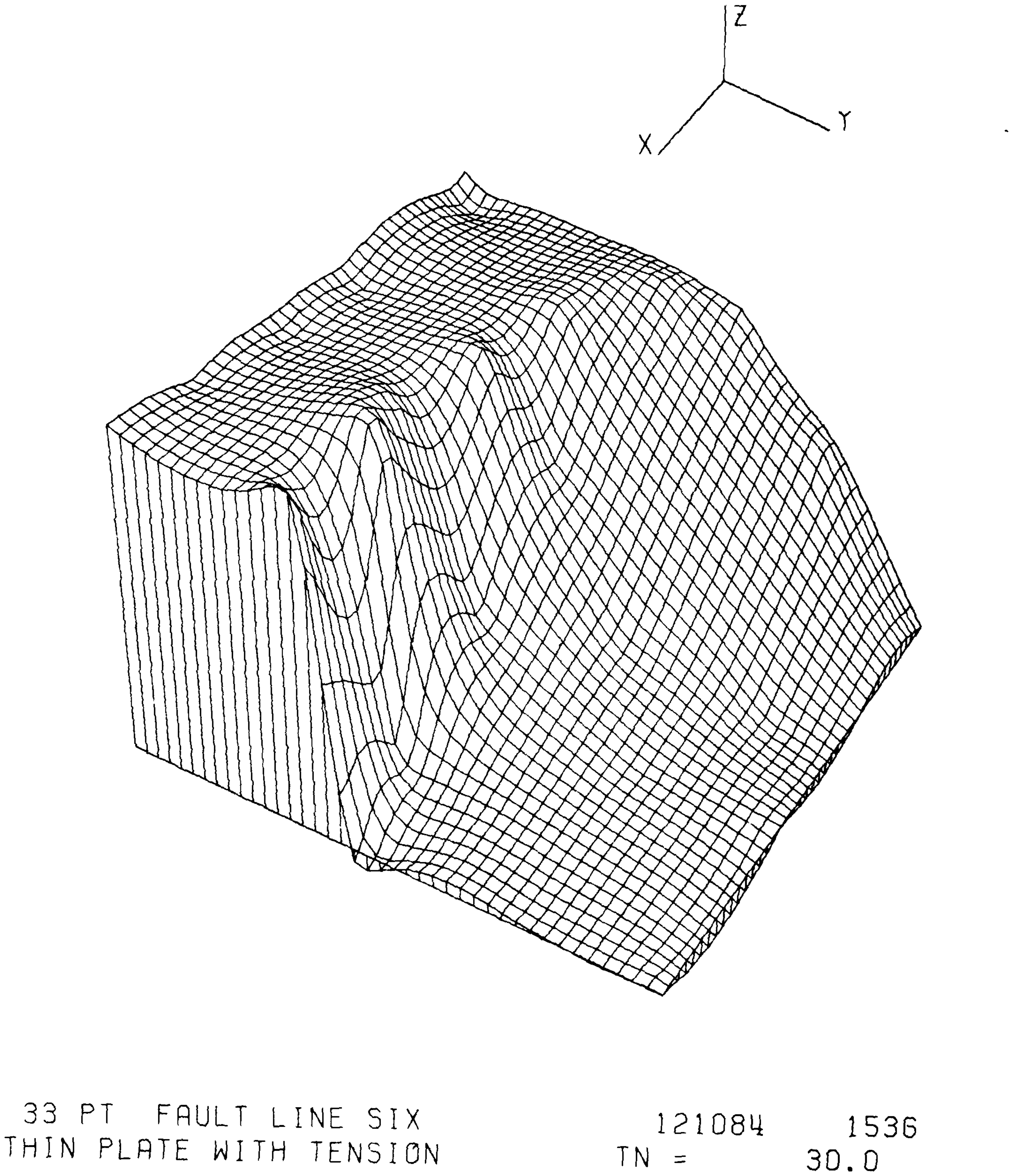

Z
X
Y
33 PT FAULT LINE SIX
THIN PLATE WITH TENSION
121084 1536
TN = 30.0

Figure 16.a

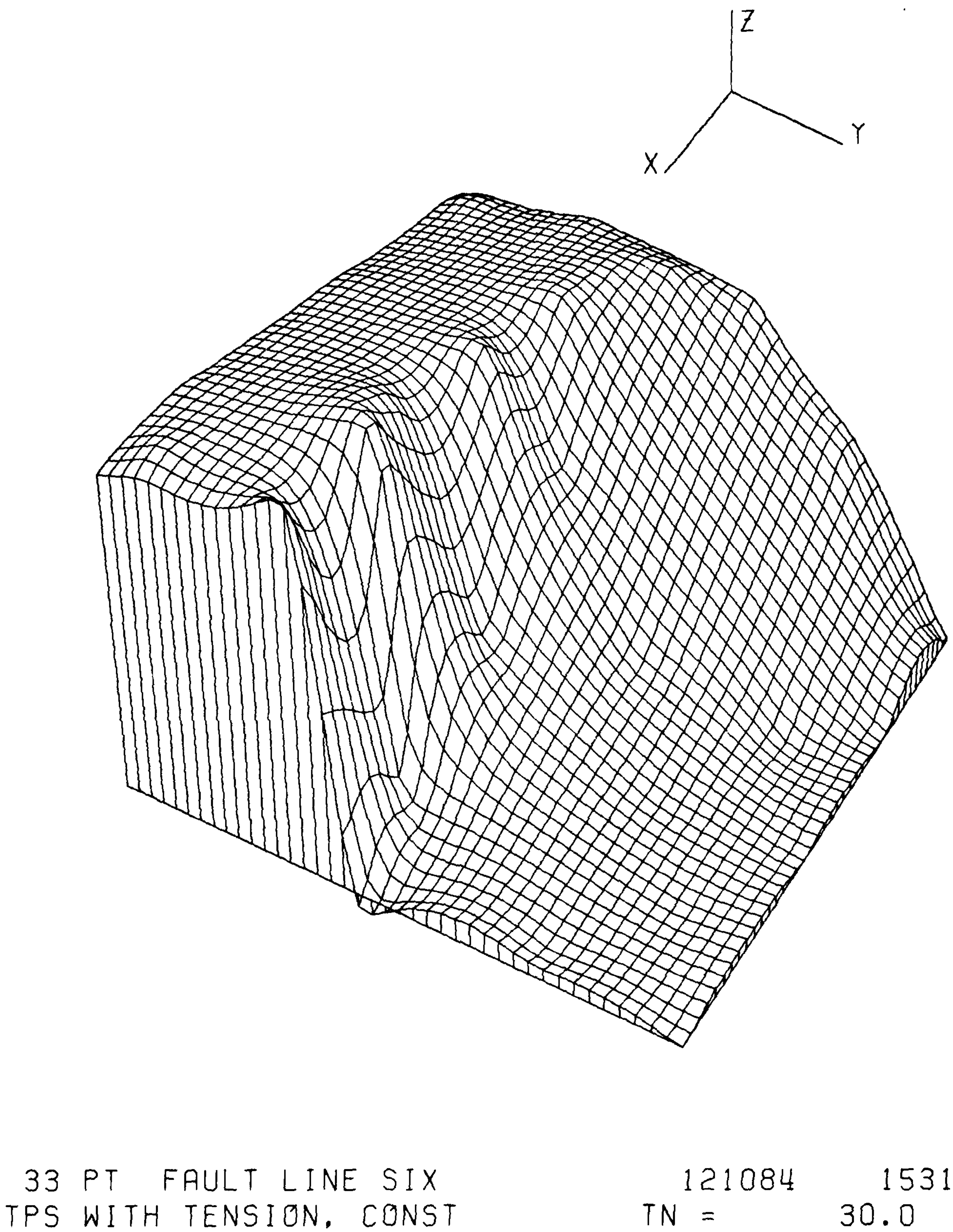

Z
Y
X
33 PT FAULT LINE SIX 121084 1531
TPS WITH TENSIØN, CØNST TN = 30.0

Figure 16.b

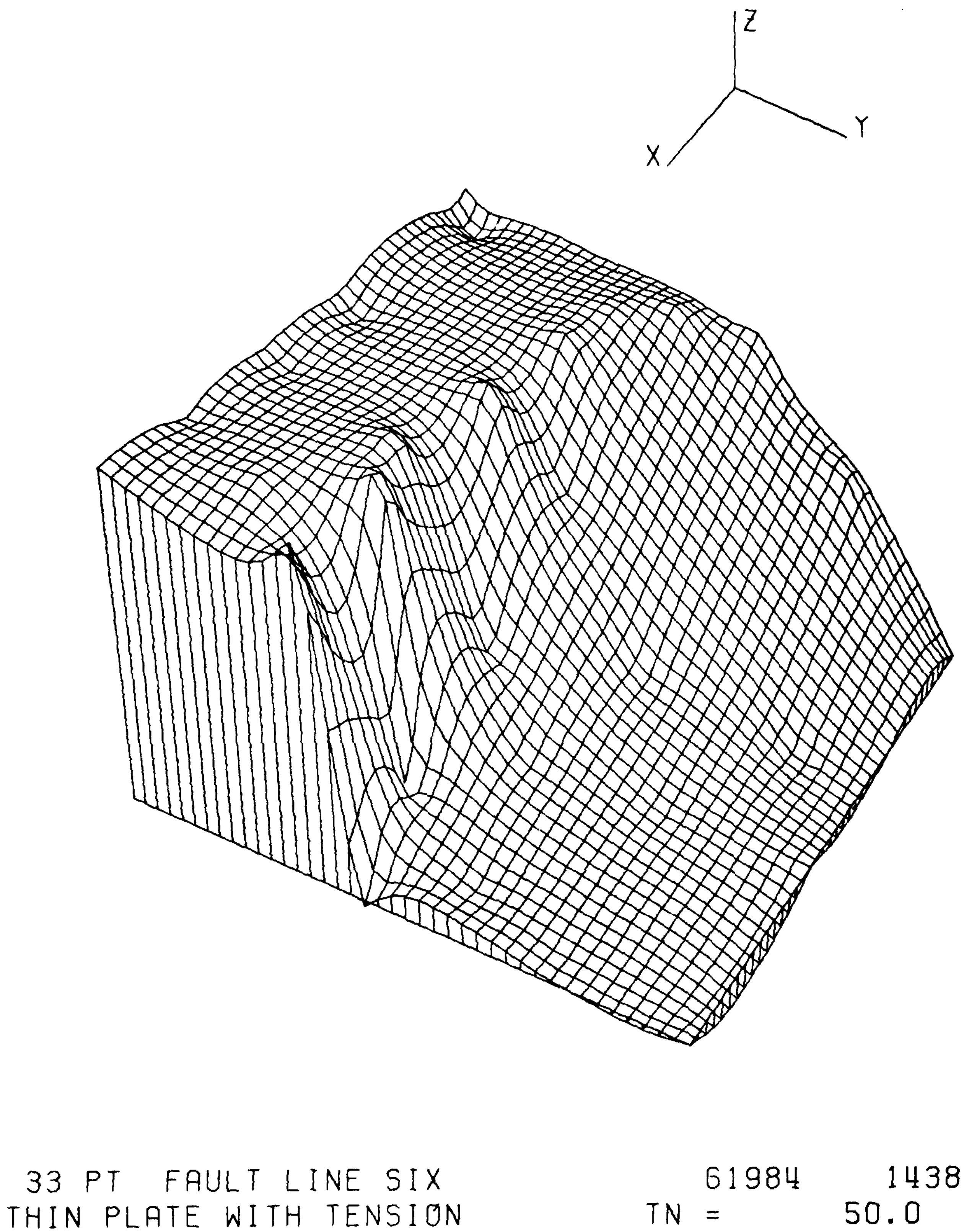
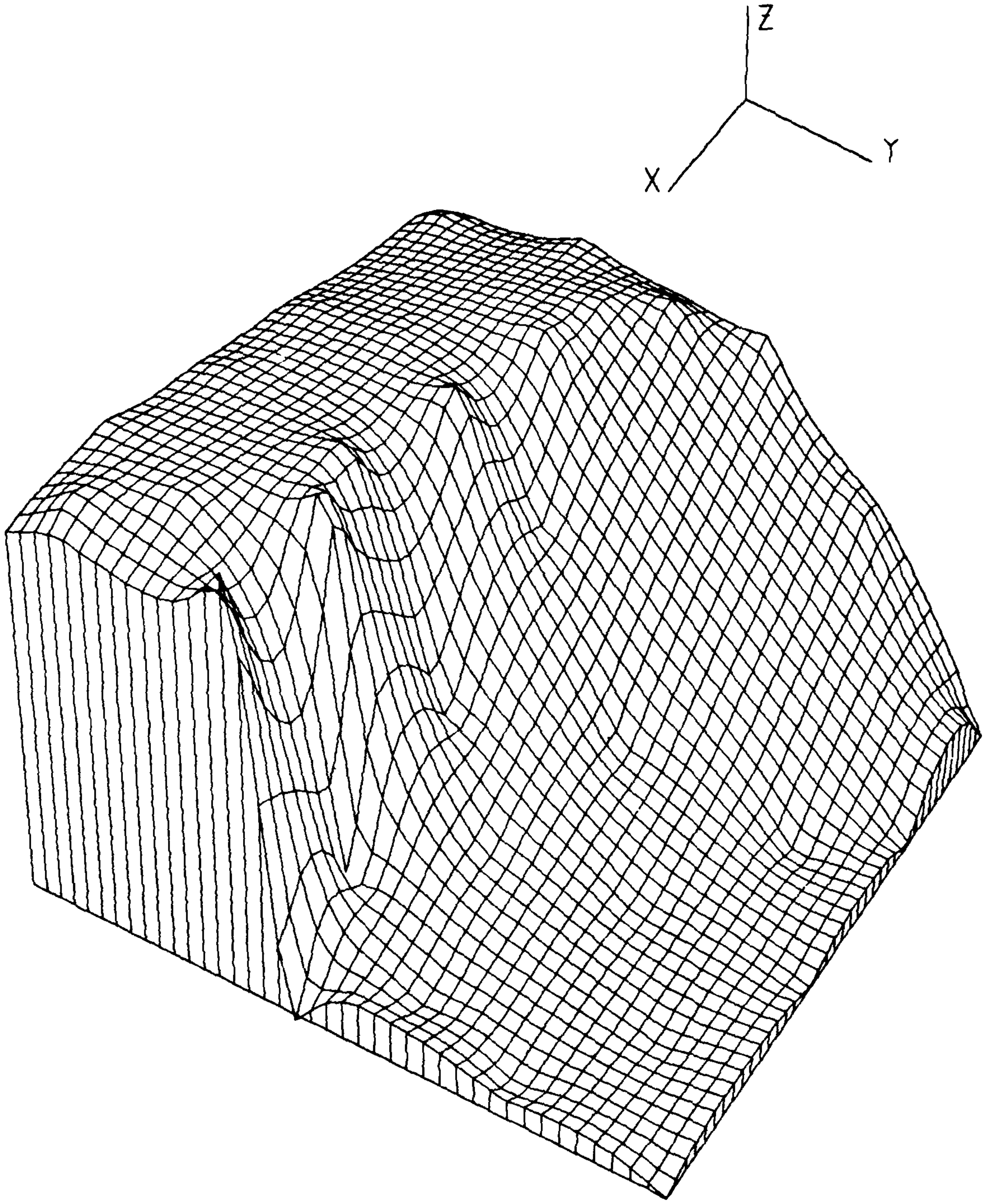

Z
Y
X
33 PT FAULT LINE SIX
THIN PLATE WITH TENSION
61984 1438
TN = 50.0

Figure 17.a



33 PT FAULT LINE SIX 113084 1233
TPS WITH TENSION, CONST TN = 50.0

Figure 17.b

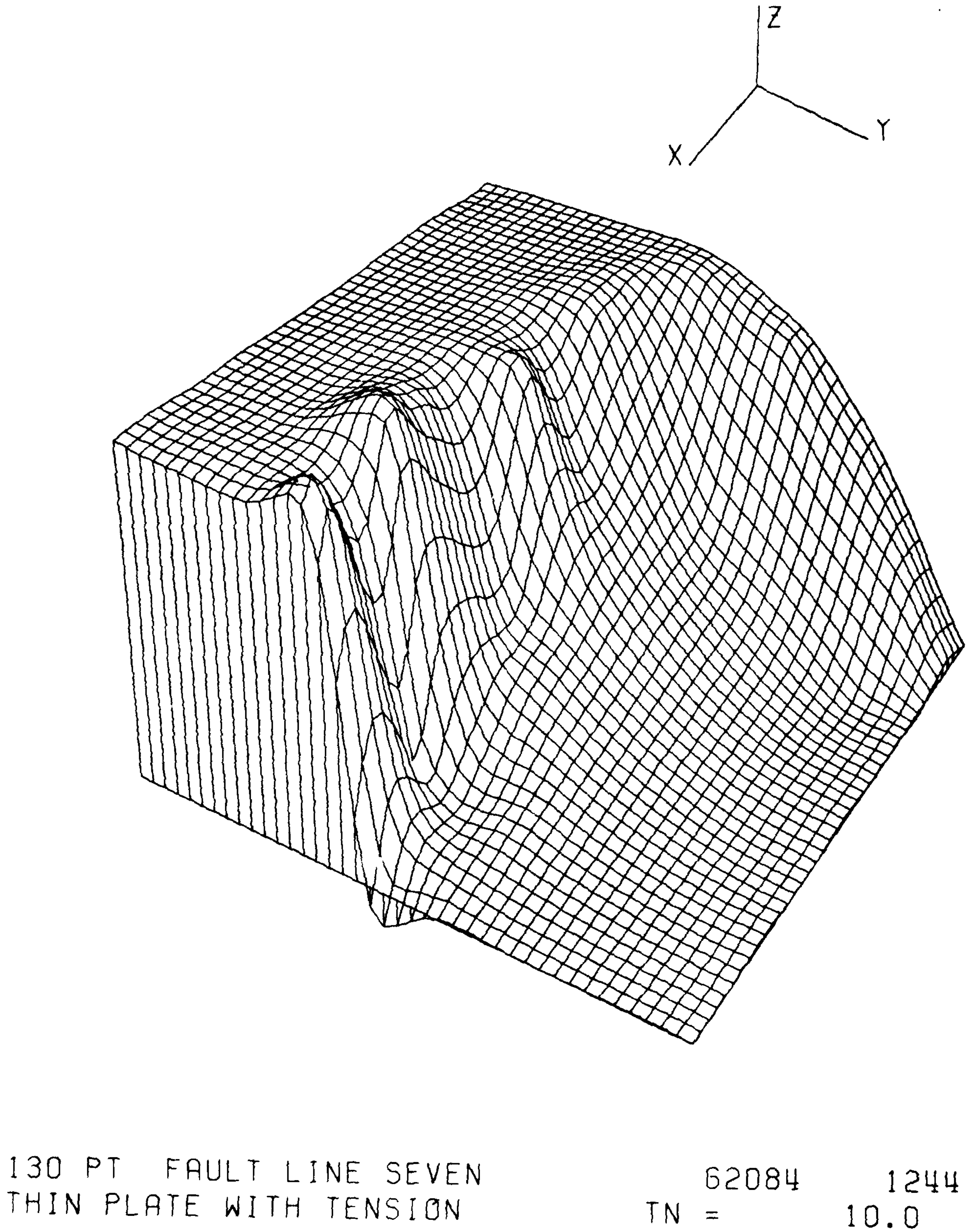

Z
X
Y
130 PT FAULT LINE SEVEN
62084 1244
THIN PLATE WITH TENSION
TN = 10.0

Figure 18.a

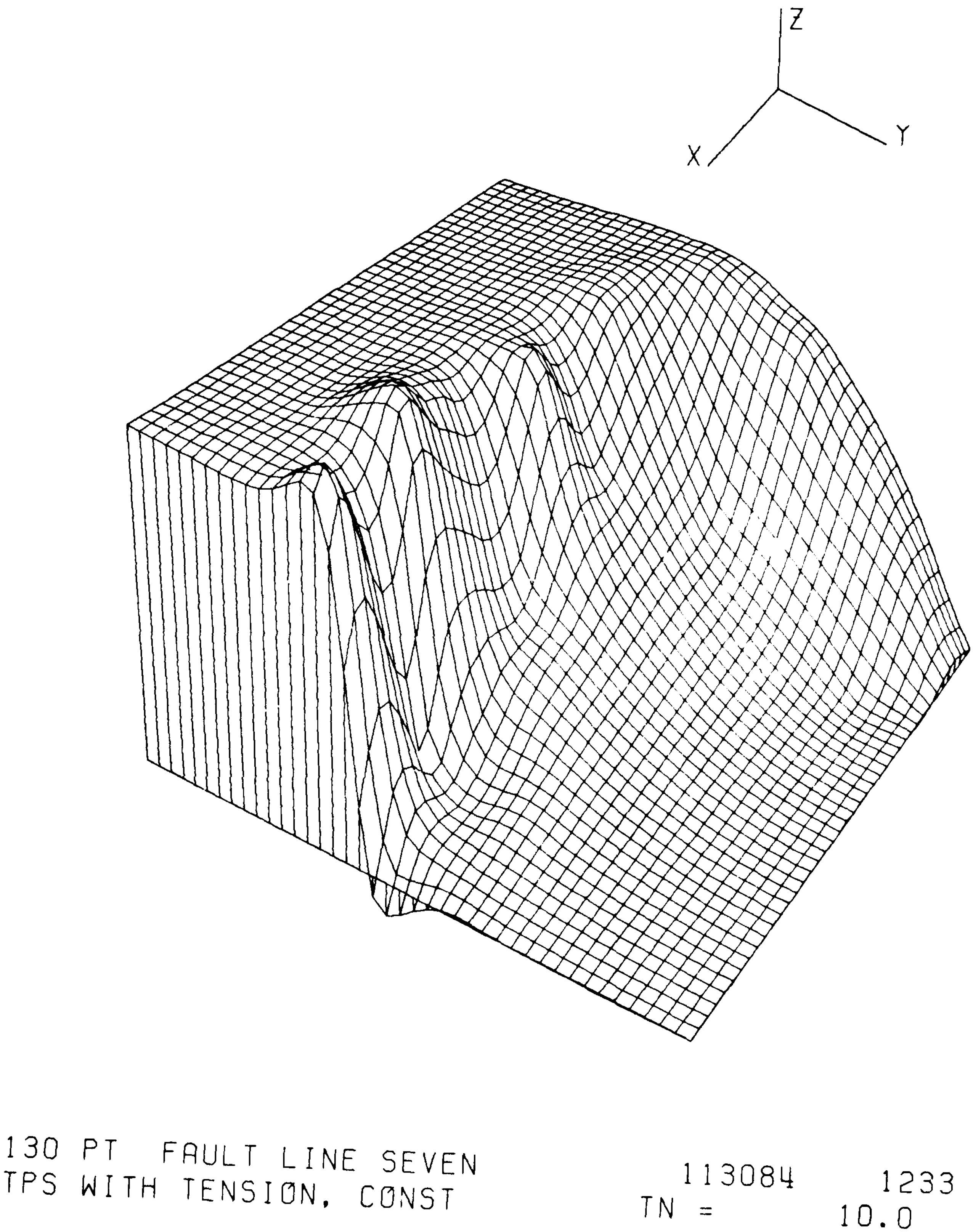
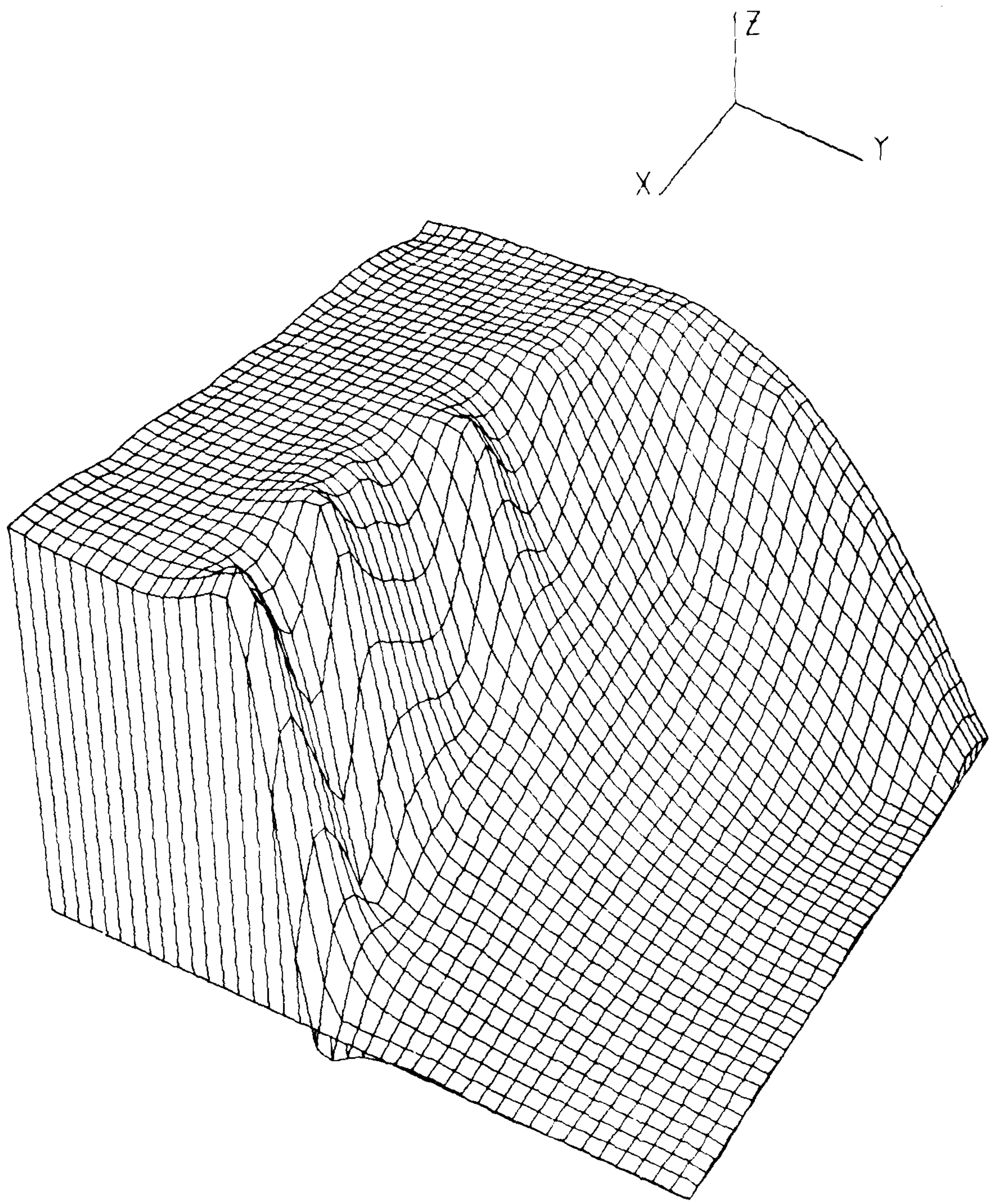
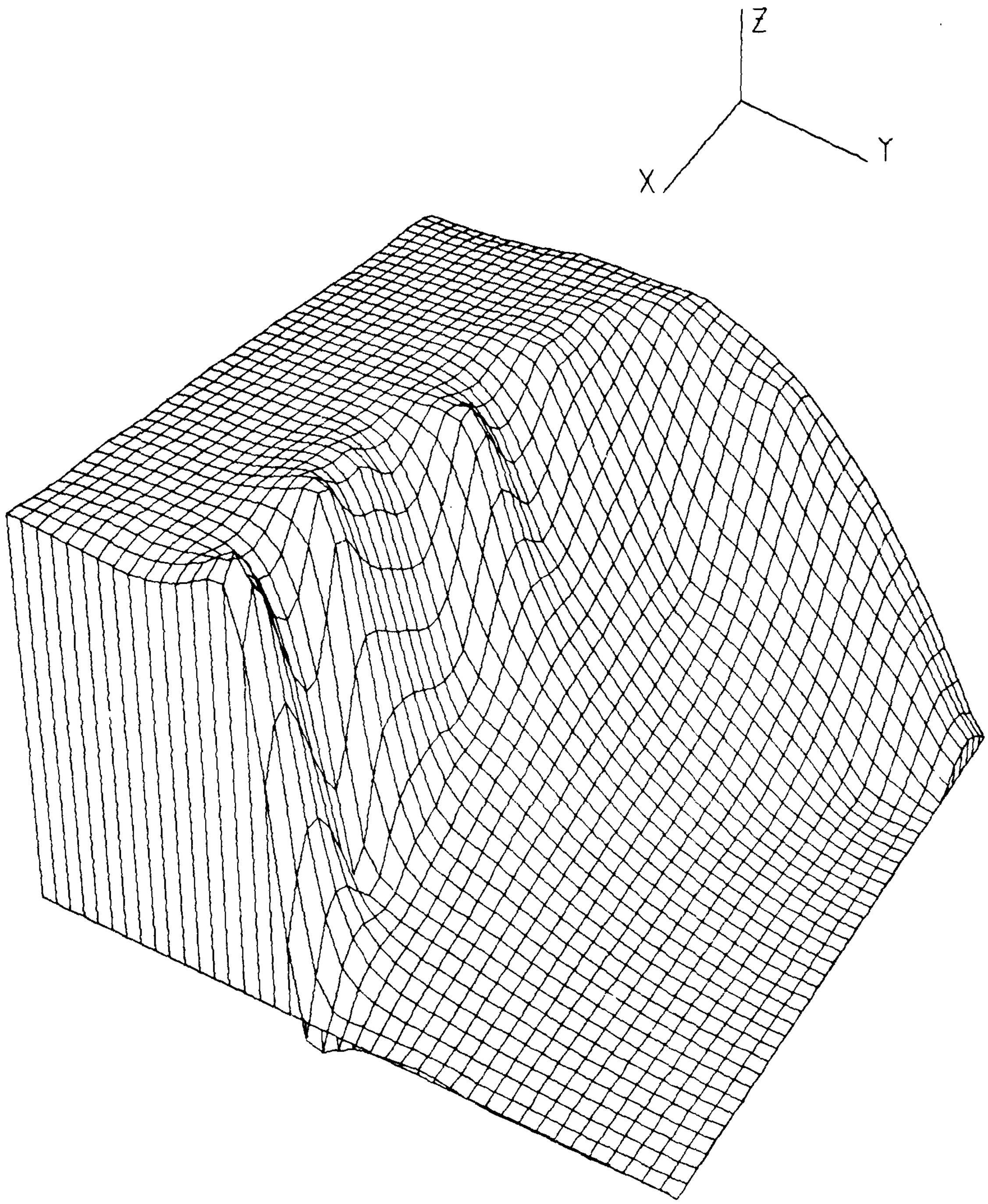

Z
X
Y
130 PT FAULT LINE SEVEN
TPS WITH TENSION, CONST
113084 1233
TN = 10.0

Figure 18.b



130 PT FAULT LINE SEVEN 121084 1536
THIN PLATE WITH TENSION TN = 30.0

Figure 19.a



130 PT   FAULT LINE SEVEN   121084   1531
TPS WITH TENSION, CONST   TN =   30.0

Figure 19.b

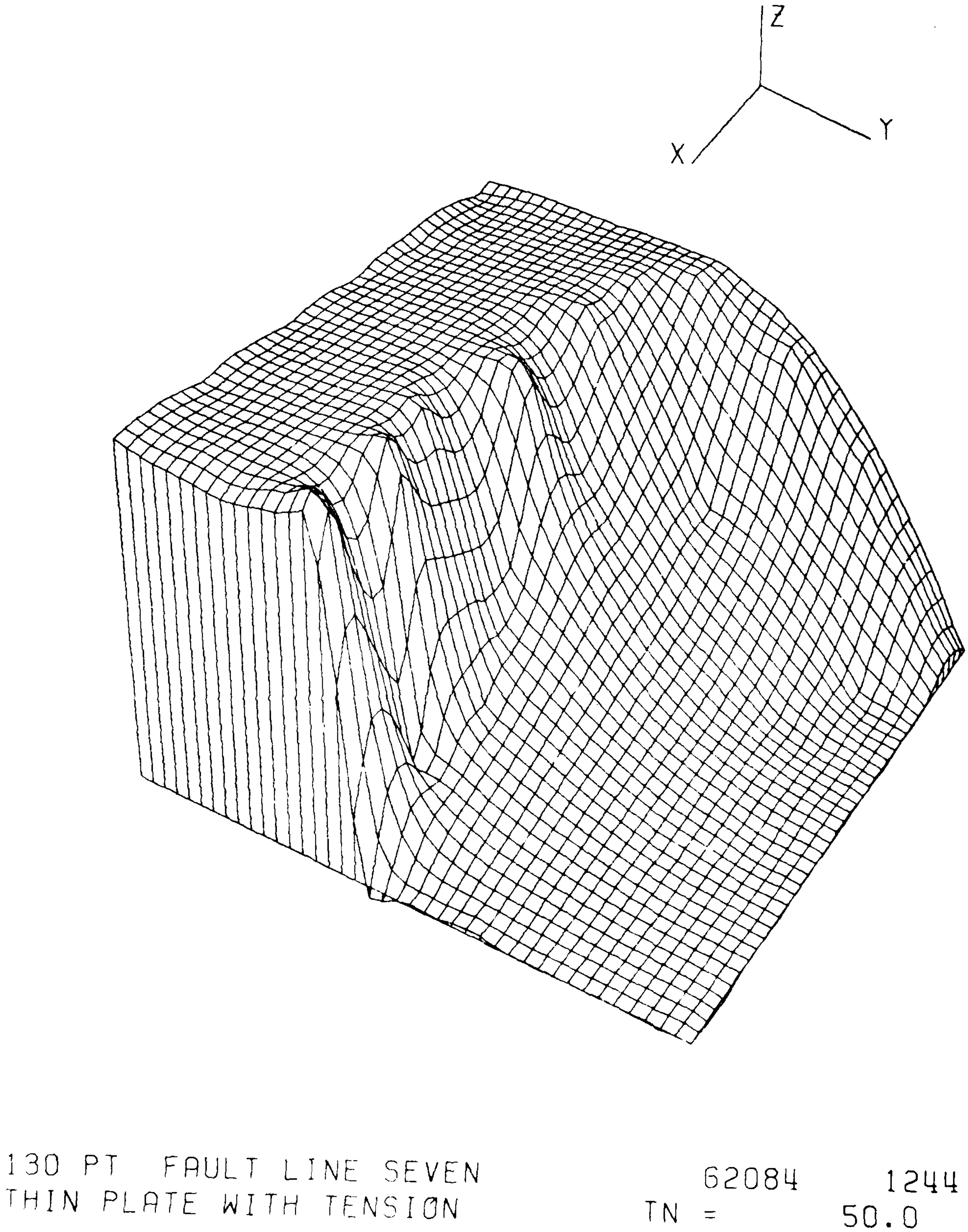
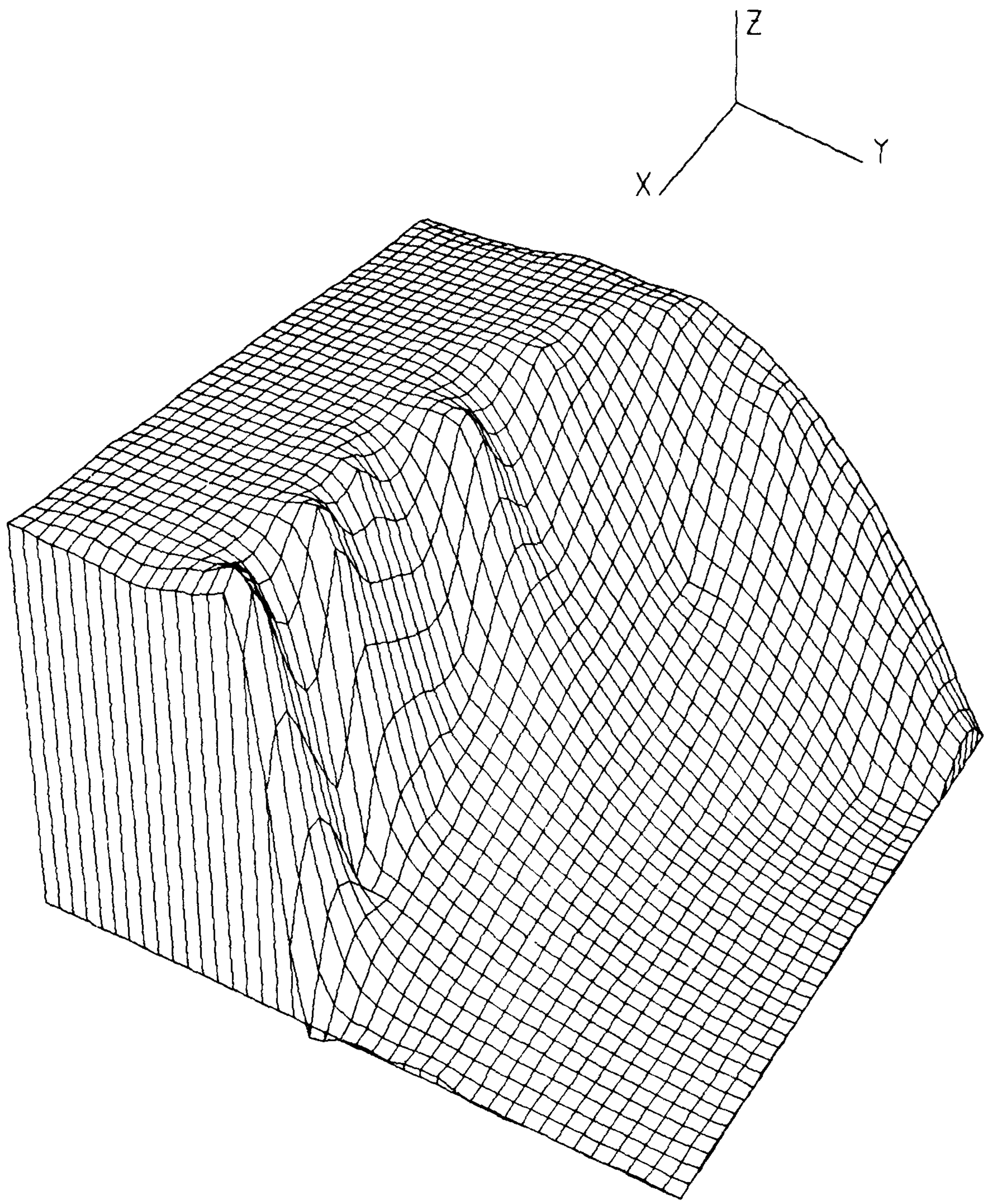

Z
X
Y
130 PT FAULT LINE SEVEN
THIN PLATE WITH TENSIØN
62084 1244
TN = 50.0

Figure 20.a



130 PT FAULT LINE SEVEN 113084 1234
TPS WITH TENSION, CONST TN = 50.0

Figure 20.b

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